nedelja informatike v6.0

Uvod u animaciju

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Matematička gimnazija

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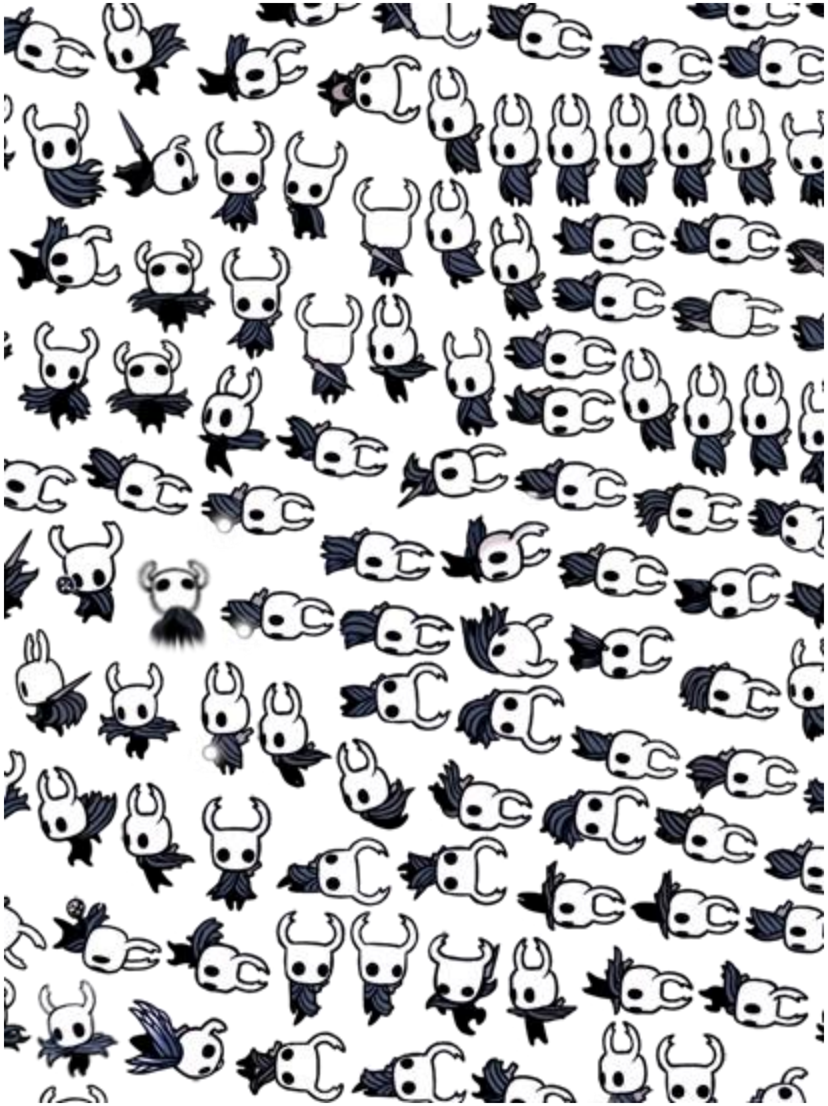
2D

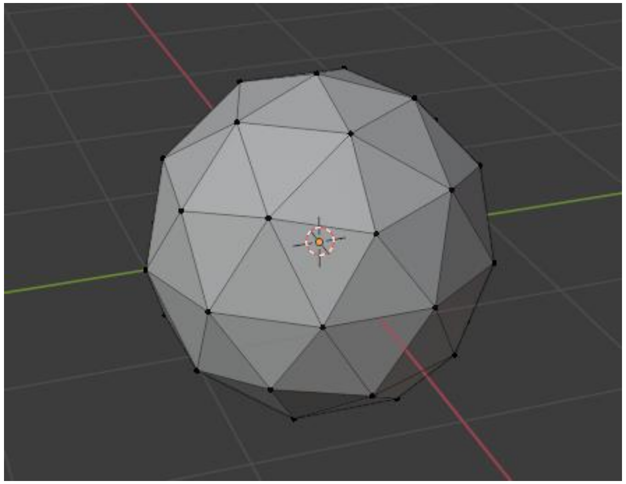


Mali uvod u 3D

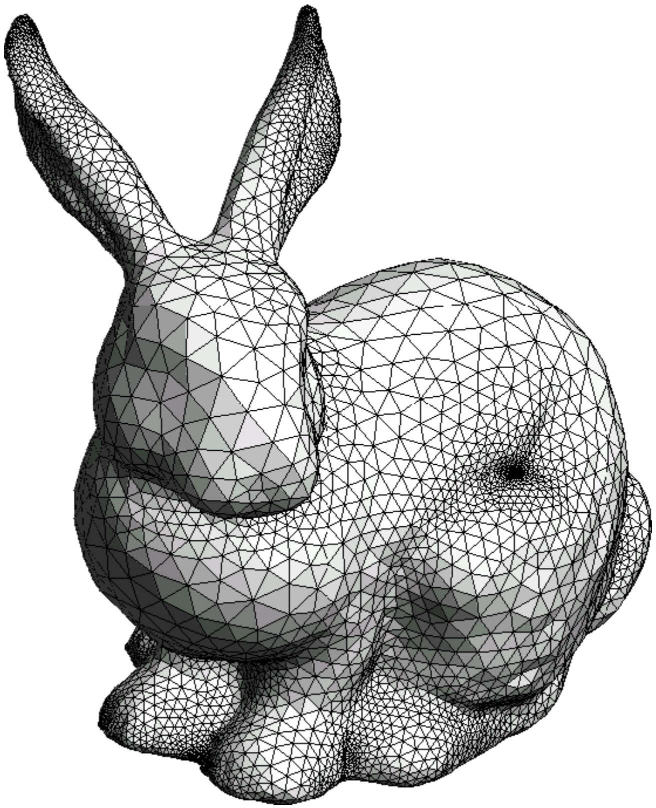
- Parametarski definisane površine
  $$f : \mathbb{R}^2 \rightarrow \mathbb{R}^3$$
- Implicitne površine i *'signed distance fields'*
  $$f(x, y, z) = 0$$
- *Point clouds*
- Poligonalne mreže

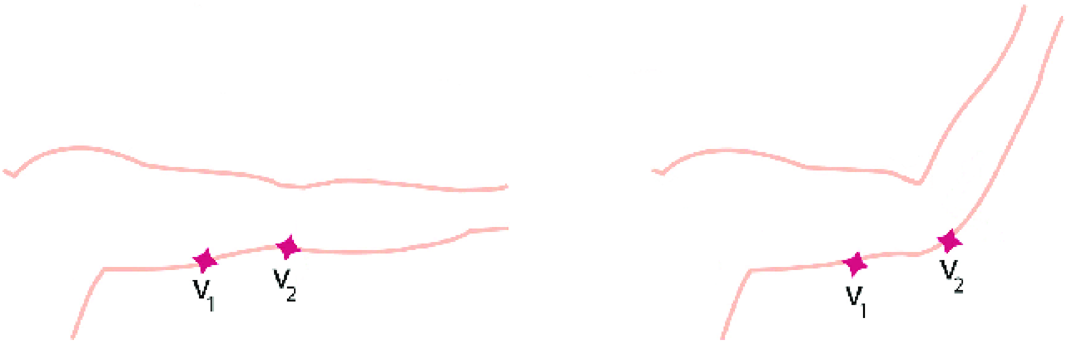
Primer: Sfera

- Parametarski: $f(\theta, \varphi) = (\cos\varphi \cos\theta, \cos\varphi \sin\theta, \sin\varphi)$
- Implicitno: $f(x, y, z) = x^2 + y^2 + z^2 - 1$
- Mreža: ???



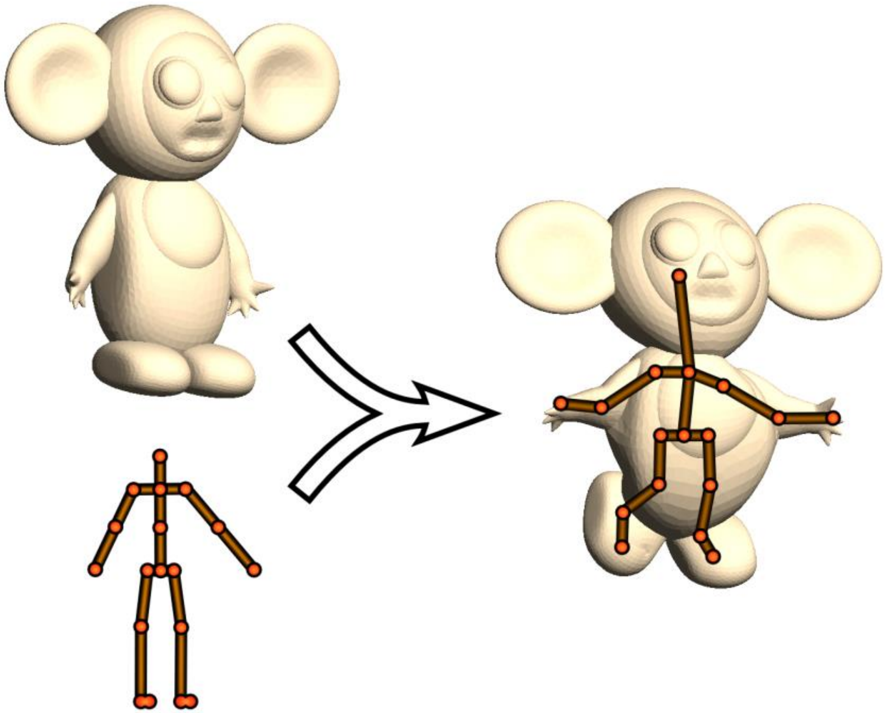
Mreža i transformacije

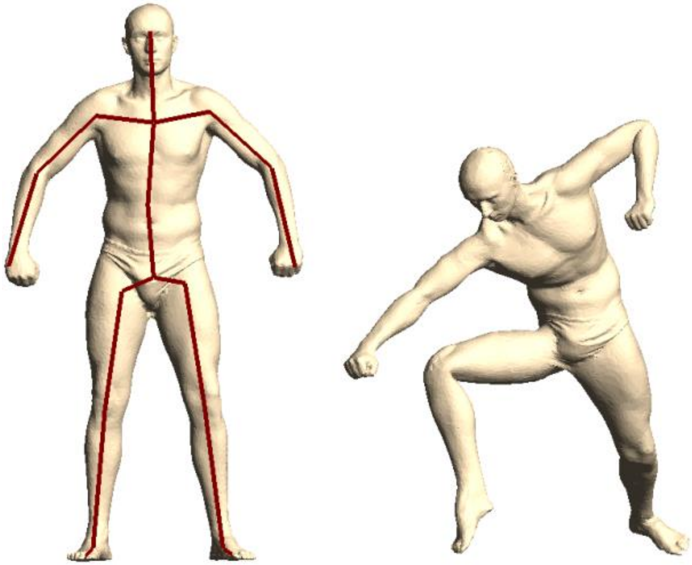




Aburumman, Nadine & Fratarcangeli, Marco. (2017). Skin Deformation Methods for Interactive Character Animation. Communications in Computer and Information Science. 8. 153-. 10.1007/978-3-319-64870-5_8.

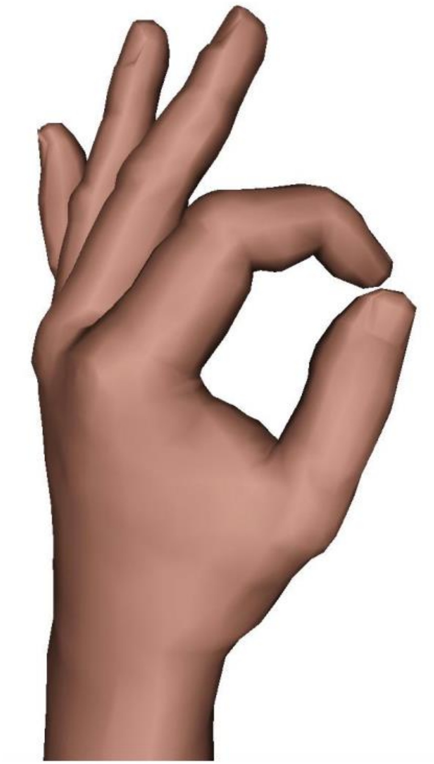
Rigging

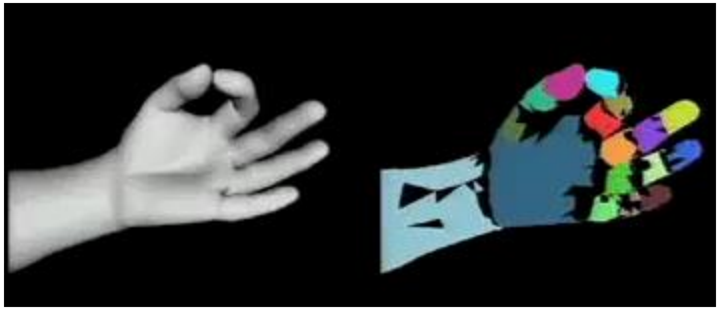




Baran, I., Popović, J. 2007. Automatic Rigging and Animation of 3D Characters. ACM Trans. Graph. 26, 3, Article 72 (July 2007), 8 pages. DOI = 10.1145/1239451.1239523
http://doi.acm.org/10.1145/1239451.1239523.

Rigging

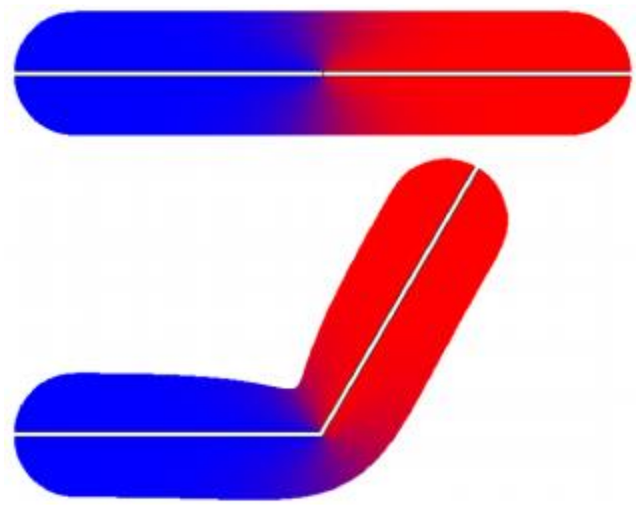




Doug L. James and Christopher D. Twigg. 2005. Skinning mesh animations. ACM Trans. Graph. 24, 3 (July 2005), 399–407. DOI: https://doi.org/10.1145/1073204.1073206

Skinning

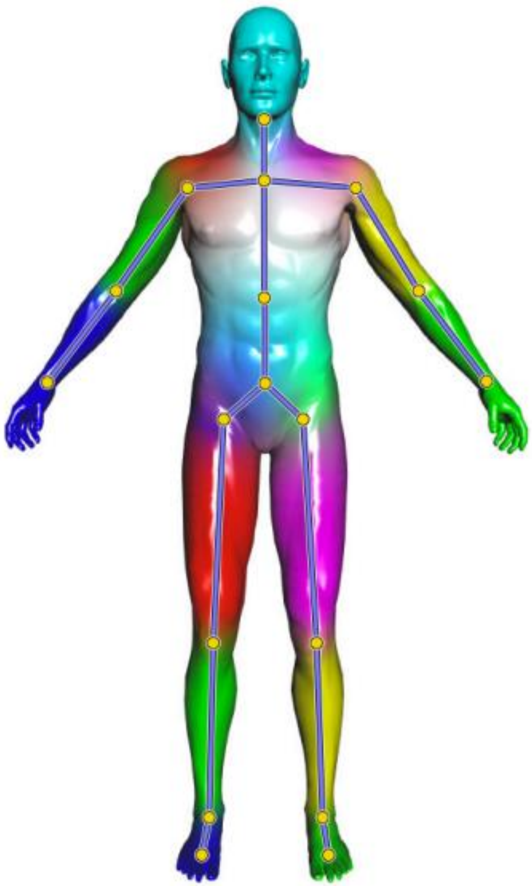
- Uparivanje mreže i skeleta
- T transformacija čvora
- T_1, T_2 transfomacije kostiju
- Ideja: Težine w_1, w_2
  - $T = avg(T_1, w_1; T_2, w_2)$
  - avg interpolira između T_1 i T_2



Baran et al

Mini Rezime

- Mreža v_i
- Skelet b_j
- Težine uticaja kosti na čvor $w_{i,j}$
- Interpolacija 2 transformacije
  - $T(v) = avg(T_1, w_1; T_2, w_2)$
  - T, T_1, T_2 su izometrije u $\mathbb{R}^3$



Linearna interpolacija

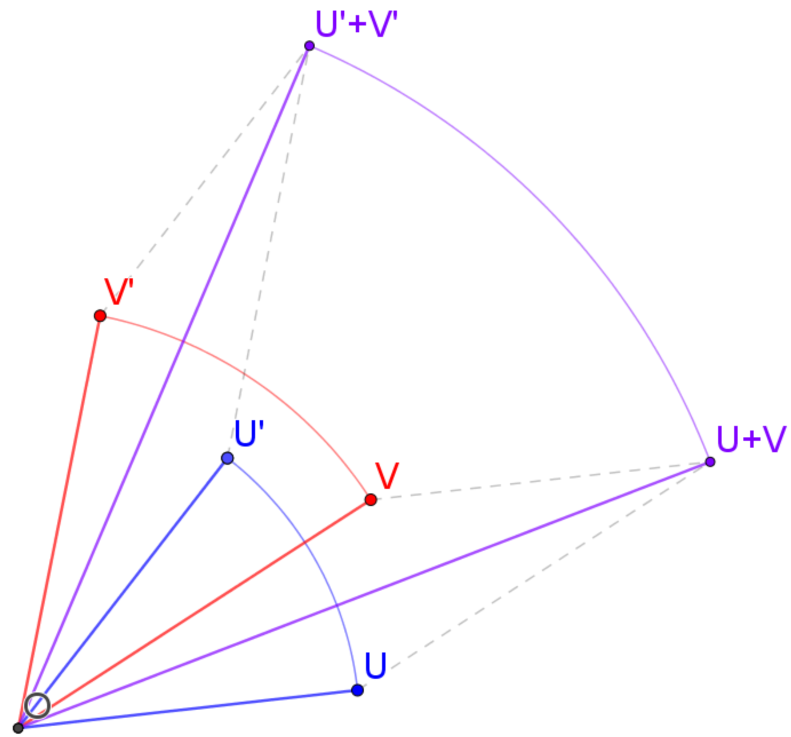
- $avg(T_1, w_1; T_2, w_2) = w_1 T_1 + w_2 T_2; \qquad w_1 + w_2 = 1$
- Generalizacija: $T(v_i) = \sum w_{i,j} T_j$
- Ali šta su uopšte transformacije?

Matrice

- Matrica $M_{n\times n}$ je linearna transformacija $\mathbb{R}^n$
- Linearnost: $M\cdot(u+v)=(M\cdot u)+(M\cdot v)$
- $M_{n\times n}=\begin{bmatrix} a_{1,1} & \cdots & a_{1,n} \\ \vdots & \ddots & \vdots \\ a_{n,1} & \cdots & a_{n,n} \end{bmatrix}$
- Množenje, transpozicija, inverz…

Matrica rotacije

- Rotacija je linearna transformacija
- Dakle, rotaciji odgovara neka matrica R
- Rotacija je izometrija
- Dakle... R je rotacija ako:
  - $R^T = R^{-1}$
  - $\det R = 1$



$$\|Rx\| = \|x\| \Rightarrow$$
$$(Rx)^T(Rx) = x^T x \Rightarrow$$
$$x^T R^T R x = x^T x \Rightarrow$$
$$R^T R = I$$

Primeri

- U 2D: $R_\theta = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$
- U 3D rotacije imaju i osu
  - $\begin{bmatrix} 0.866 & 0 & 0.5 \\ 0 & 1 & 0 \\ -0.5 & 0 & 0.866 \end{bmatrix}$ je rotacija u pravcu y ose za 30°
  - $\begin{bmatrix} 0.888 & -0.385 & 0.248 \\ 0.430 & 0.888 & -0.159 \\ -0.159 & 0.248 & 0.955 \end{bmatrix}$ je rotacija u pravcu $(1{,}1{,}2)$ za 30°

Translacija

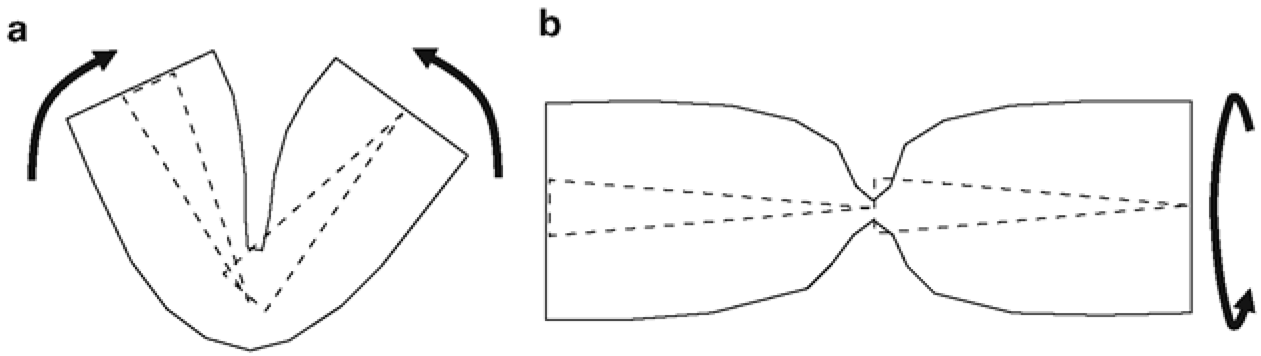
- Nije linearna transformacija!
- Bar ne u 3D...

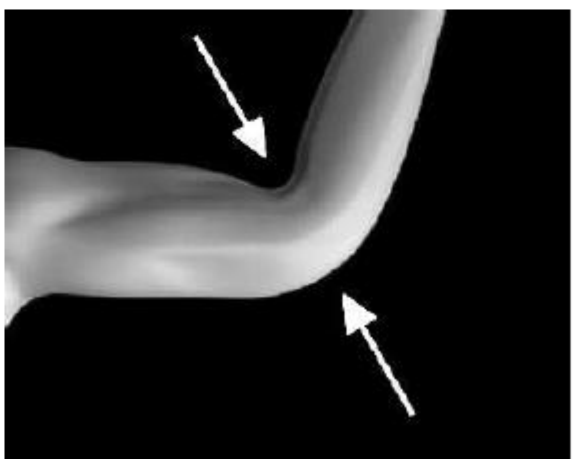
Homogene 4D koordinate

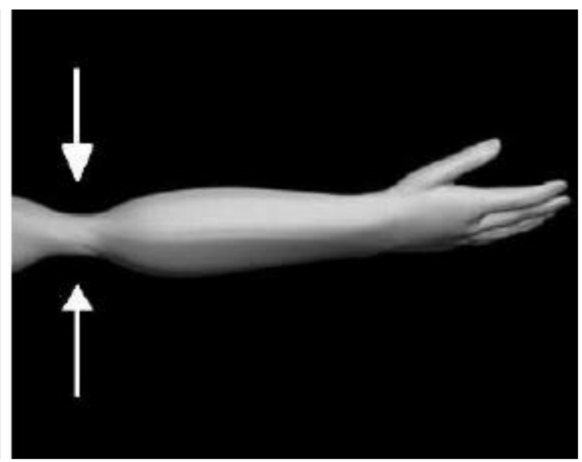
- Nov koordinatni sistem: $(x, y, z, w) \in \mathbb{R}^4$
- $(x, y, z, w) \leftrightarrow \left(\frac{x}{w}, \frac{y}{w}, \frac{z}{w}\right), w \neq 0$
- Rotacija je 4D matrica : $\begin{bmatrix} & & & 0 \\ & \boldsymbol{R} & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
- Translacija je 4D matrica: $\begin{bmatrix} 1 & 0 & 0 & v_x \\ 0 & 1 & 0 & v_y \\ 0 & 0 & 1 & v_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Linear Blend Skinning

$$T = w_1 T_1 + w_2 T_2$$



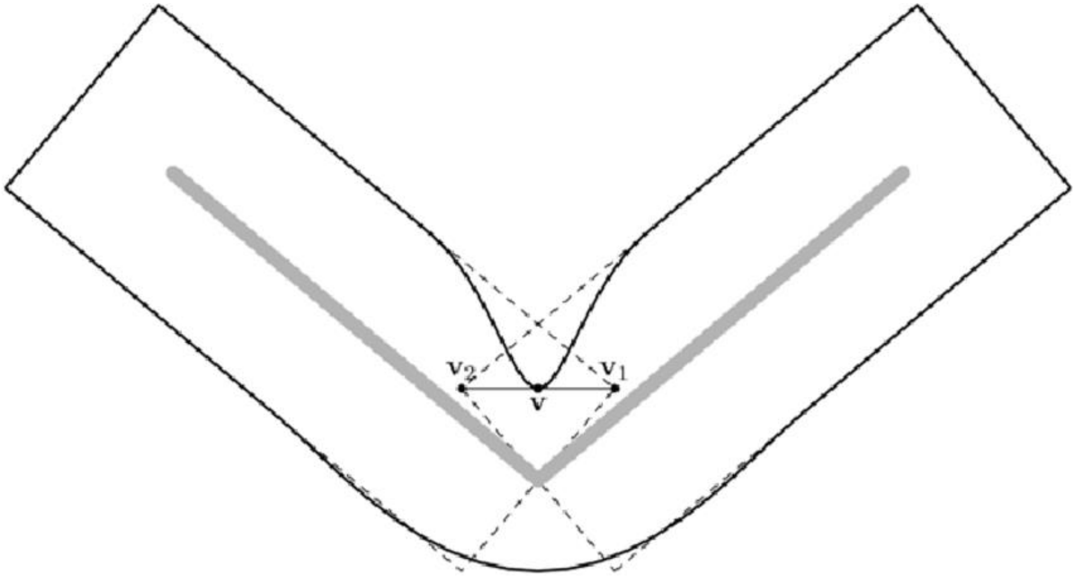




Elbow dip

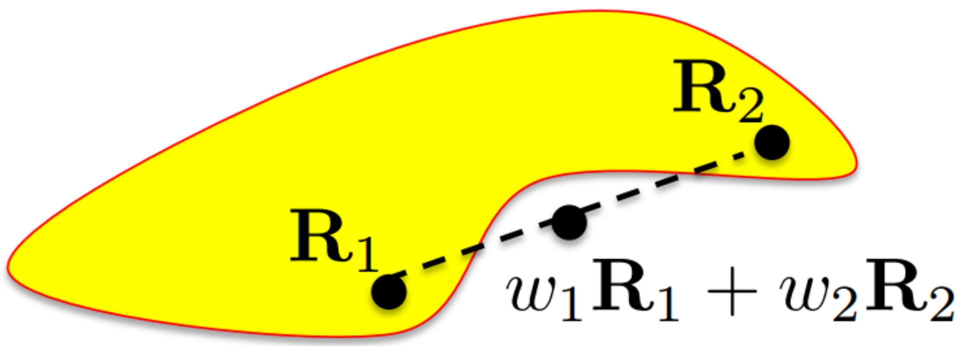
Candy wrapper

Linear Blend Skinning



Linear Blend Skinning

- Matematički uzrok
- $w_1 \tau_1 + w_2 \tau_2$ je translacija „slučajno"
- $w_1 R_1 + w_2 R_2$ nije rotacija!
- Sabiranje matrica nema geometrijski smisao



Linear Blend Skinning

- Drugačija funkcija interpolacije avg
- Popravljena linearna interpolacija ($f\colon\ w_1 R_1 + w_2 R_2 \ \longmapsto R$)
- Nova parametrizacija rotacije

Kvaternioni $\mathbb{H}$

$\mathbb{C}$	$\mathbb{H}$
$z = a + bi$	$\boldsymbol{q} = a + bi + cj + dk$
„Sadrži“ 2 realna broja	„Sadrži“ 4 realna broja
$\Re(z) = a$	$Sc(\boldsymbol{q}) = a$
$\Im(z) = b$	$Vec(\boldsymbol{q}) = bi + cj + dk$
Sabira se po komponentama	Sabira se po komponentama
$\bar{z} = a - bi$	$\boldsymbol{q}^* = a - bi - cj - dk$
$i^2 = -1$	*Videćemo...*
Predstavlja rotaciju u 2D	*Predstavlja rotaciju u 3D...*

Množenje kvaterniona

$\mathbb{C}$	1	i
1	1	i
i	i	-1

$\mathbb{H}$	1	i	j	k
1	1	i	j	k
i	i	-1	$\boldsymbol{k}$	$-\boldsymbol{j}$
j	j	$-\boldsymbol{k}$	-1	$\boldsymbol{i}$
k	k	$\boldsymbol{j}$	$-\boldsymbol{i}$	-1

- $i^2 = j^2 = k^2 = ijk = -1$
- Množenje više nije komutativno
$$\boldsymbol{q}_1\boldsymbol{q}_2 \neq \boldsymbol{q}_2\boldsymbol{q}_1$$
- Ali za čisto vektorske kvaternione $\boldsymbol{q}_1, \boldsymbol{q}_2$:
$$Vec(\boldsymbol{q}_1\boldsymbol{q}_2) = -Vec(\boldsymbol{q}_2\boldsymbol{q}_1)$$
- $u \times v = -v \times u$
- Slučajnost?

Kvaternioni i geometrija

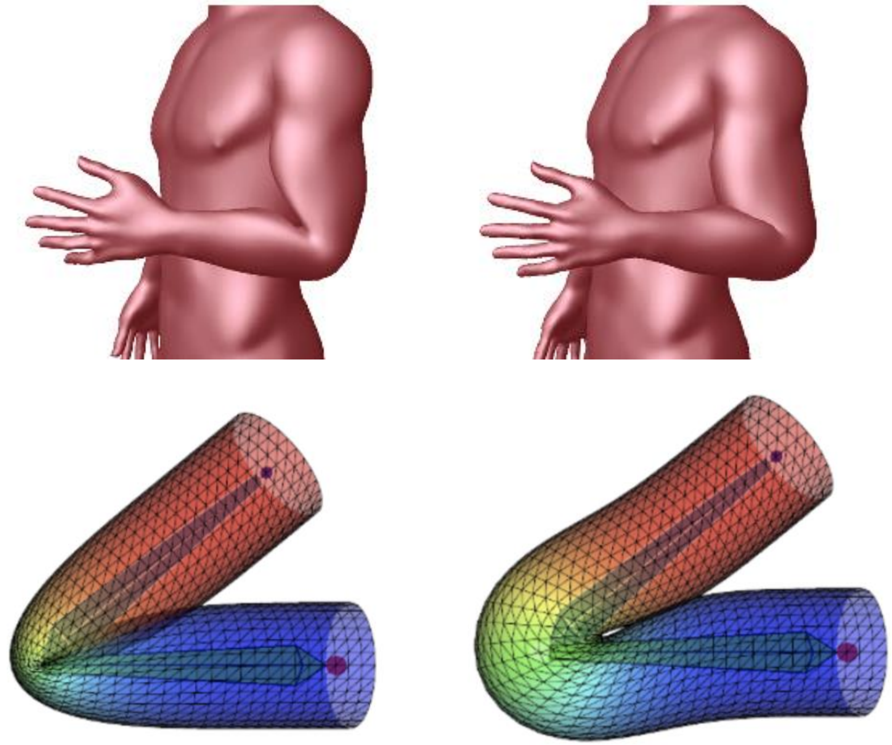
- $(x, y, z) \leftrightarrow xi + yj + zk$
- Jedinični kvaternion $\boldsymbol{q} = \cos\frac{\theta}{2} + \boldsymbol{s}\sin\frac{\theta}{2}; \;\; \boldsymbol{s} = xi + yj + zk, \|\boldsymbol{s}\| = 1$
- $\boldsymbol{qvq}^*$ je rotacija za θ u pravcu $\boldsymbol{s}$
- Translacija?
- Postoji način...
- Dualni kvaternion: $\widehat{\boldsymbol{q}} = \boldsymbol{q}_0 + \varepsilon\boldsymbol{q}_\varepsilon; \;\; \varepsilon^2 = 0$
- Jedinični dualni kvaternion kodira izometrije u 3D

Dual Quaternion Skinning

$$\widehat{\boldsymbol{q}} = w_1 \widehat{\boldsymbol{q}}_1 + w_2 \widehat{\boldsymbol{q}}_2?$$

Dual Quaternion Skinning

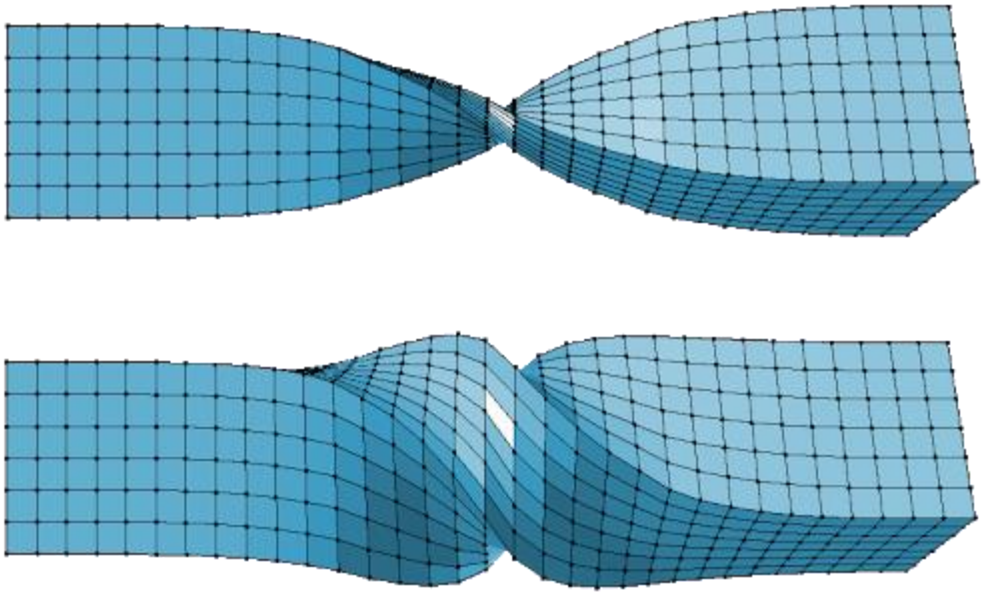
$$\hat{\boldsymbol{q}} = \frac{w_1\hat{\boldsymbol{q}}_1 + w_2\hat{\boldsymbol{q}}_2}{\|w_1\hat{\boldsymbol{q}}_1 + w_2\hat{\boldsymbol{q}}_2\|}$$



LBS DQS

Dual Quaternion Skinning

$$\hat{\boldsymbol{q}} = \frac{w_1\hat{\boldsymbol{q}}_1 + w_2\hat{\boldsymbol{q}}_2}{\|w_1\hat{\boldsymbol{q}}_1 + w_2\hat{\boldsymbol{q}}_2\|}$$



Zakjučak

- Koristili smo skelet da pojednostavimo animacije
- LBS i DQS su najkorišćenije tehnike
- Mnoge stvari nismo spomenuli
  - Motion Capture
  - Alternative za DQS (https://www.youtube.com/watch?v=DfIfcQiC2oA)
  - Razne forice (keyframing, particles...)