

Neuralne mreže kao univerzalni aproksimatori

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Matematička gimnazija

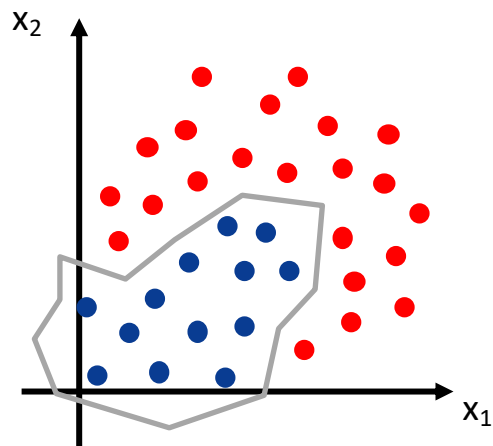
27. 04. 2021.

- Govorićemo o:
 - Šta su neuralne mreže (potpuno povezane)
 - Kako klasifikuju i aproksimiraju
 - Postoji li mreža koja klasifikuje/aproksimira kako želimo?
- Nećemo govoriti o:
 - Kako naučiti mrežu da pravilno klasifikuje/aproksimira
 - Konvolutivne mreže, GAN, LSTM,...
- Napomena: svi primeri u 2D

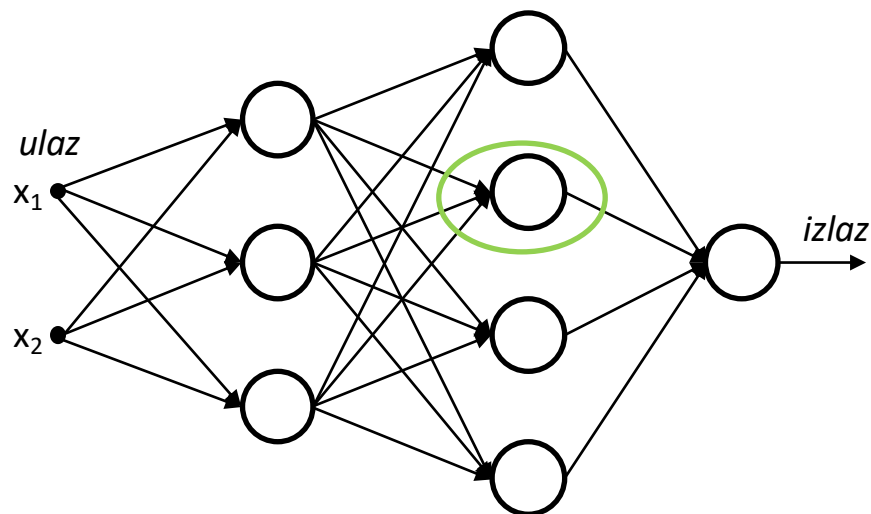
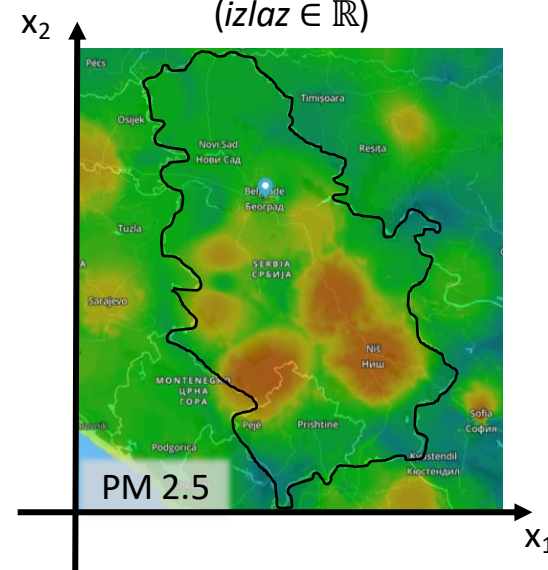
Neuralne mreže, klasifikacija, aproksimacija



Klasifikacija
(izlaz $\in \{\text{plava, crvena}\}$)



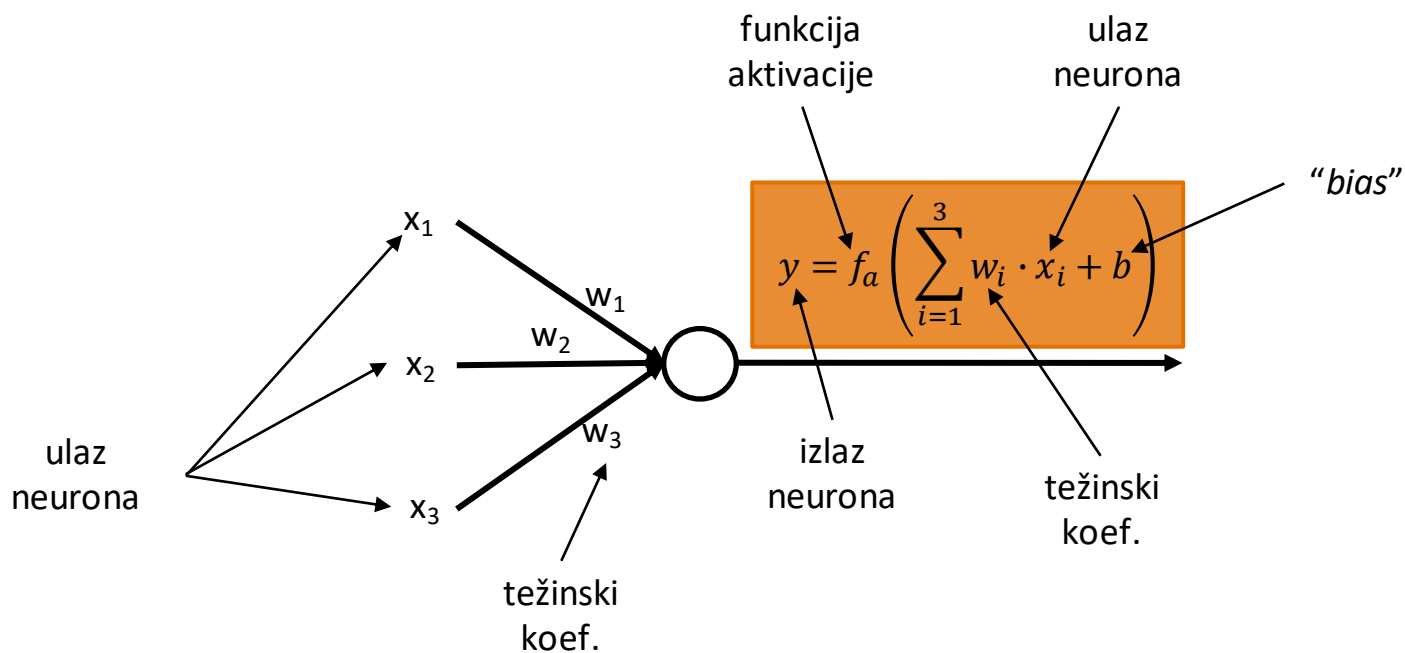
Aproksimacija (regresija)
(izlaz $\in \mathbb{R}$)



Neuron

- ~~Ćelija~~ je osnovna jedinica građe i funkcije svih živih bića

neuralnih mreža

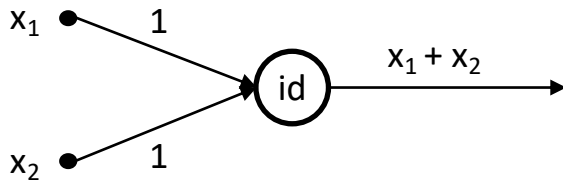


- Neuralna mreža „uči“ prilagođavanjem koeficijenata w_i i b

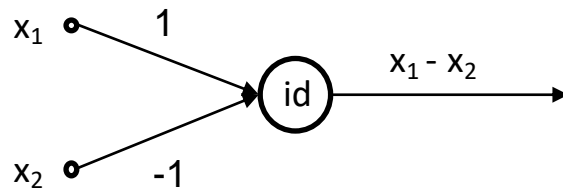
Neuron bez aktivacije



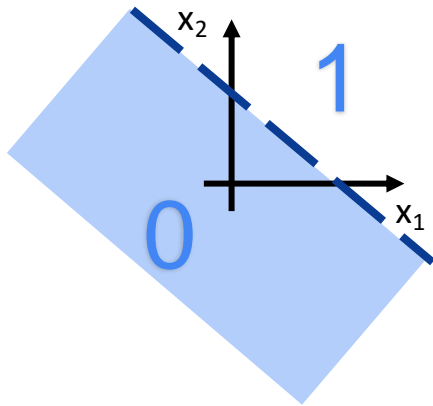
- $f_a(\sum w_i \cdot x_i + b) \stackrel{\text{def}}{=} \sum w_i \cdot x_i + b$
- Neuron kao operacija „+“:



- Neuron kao operacija „-“:

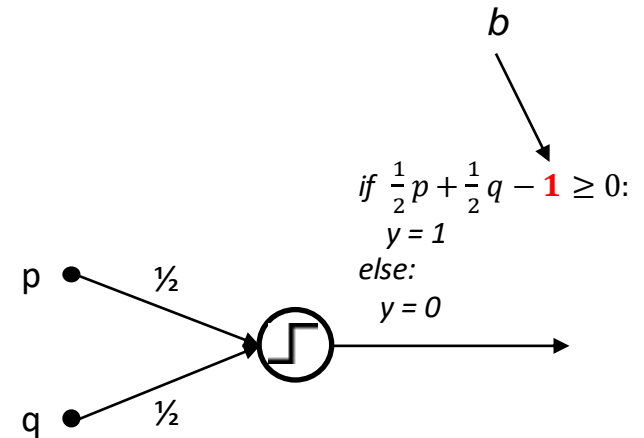
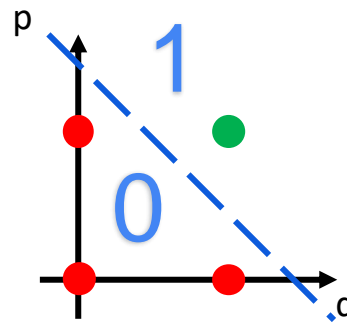


- $f_a(\sum w_i \cdot x_i + b) \stackrel{\text{def}}{=} \begin{cases} 0, & \sum \dots < 0 \\ 1, & \sum \dots \geq 0 \end{cases}$
- U 2D slučaju:
 - $w_1 x_1 + w_2 x_2 + b < 0 \quad / -w_1 x_1 - b \Leftrightarrow$
 - $w_2 x_2 < -w_1 x_1 - b \quad / : w_2 \Leftrightarrow$
 - $x_2 < -\frac{w_1}{w_2} x_1 - \frac{b}{w_2}$



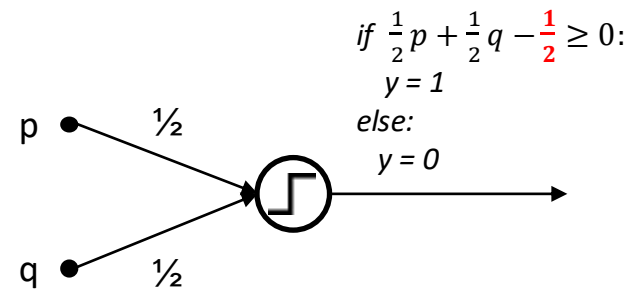
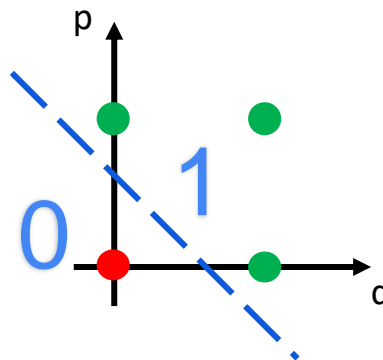
- $f_a(\sum w_i \cdot x_i + b) \stackrel{\text{def}}{=} \begin{cases} 0, & \sum \dots < 0 \\ 1, & \sum \dots \geq 0 \end{cases}$
- Neuron kao „i“ funkcija:

p	q	p AND q
0	0	0
0	1	0
1	0	0
1	1	1



- Neuron kao „ili“ funkcija:

p	q	p OR q
0	0	0
0	1	1
1	0	1
1	1	1

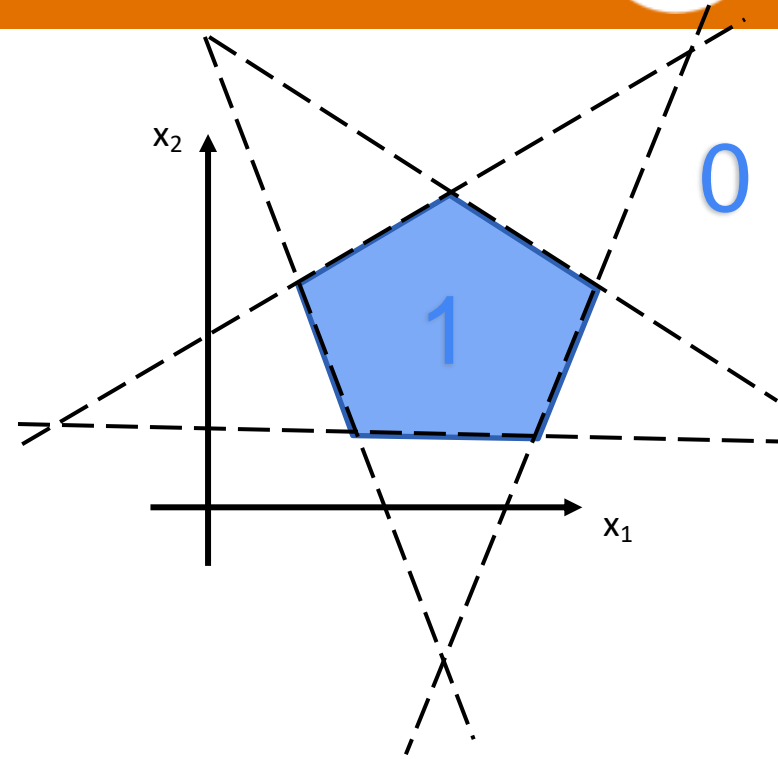
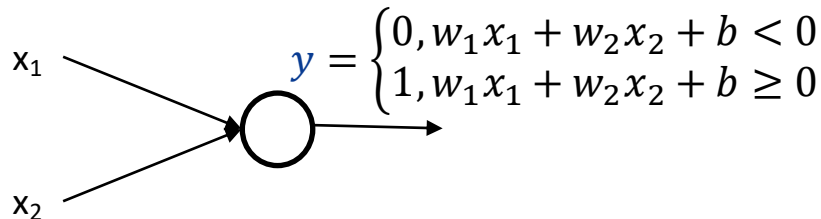
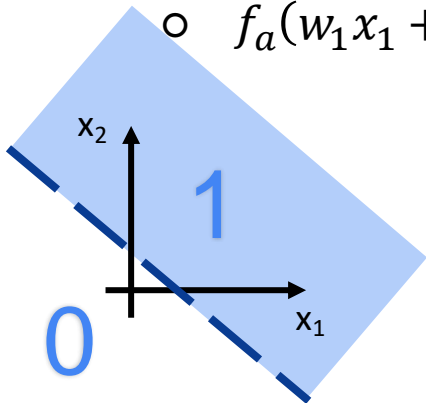


Klasifikacija (tačka u ravni)

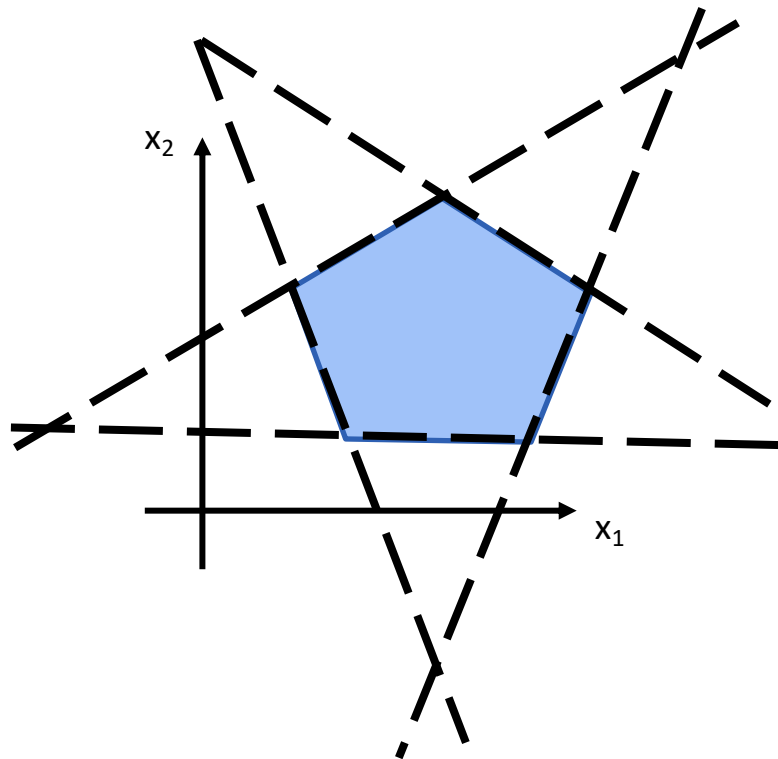


- Dva ulaza: x_1 i x_2 , realni brojevi
- Jedan izlaz: y , 0 ili 1
- Osnovno sredstvo za klasifikaciju: *prava*
- Jednačina prave: $y = kx + n$
- Opšti oblik: $ax + by + c = 0$
- Ili našim oznakama: $w_1x_1 + w_2x_2 + b = 0$
- Biramo pogodnu f-ju aktivacije za neurone:

$$f_a(w_1x_1 + w_2x_2 + b) \stackrel{\text{def}}{=} \begin{cases} 0, & \sum \dots < 0 \\ 1, & \sum \dots \geq 0 \end{cases}$$



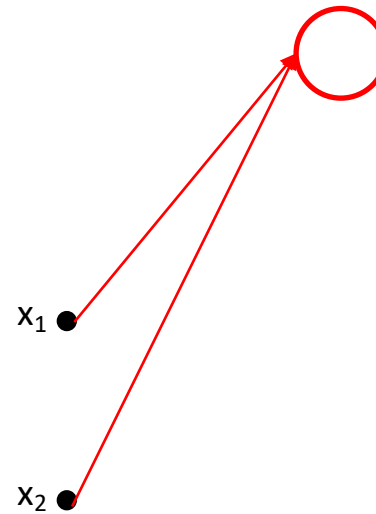
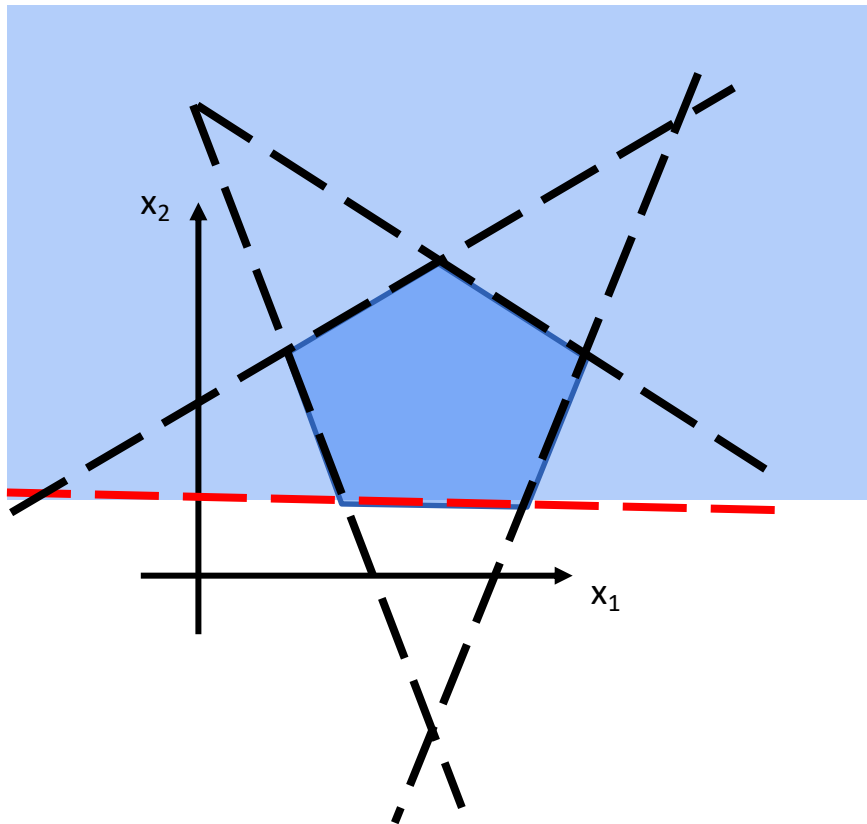
Primer: petougao



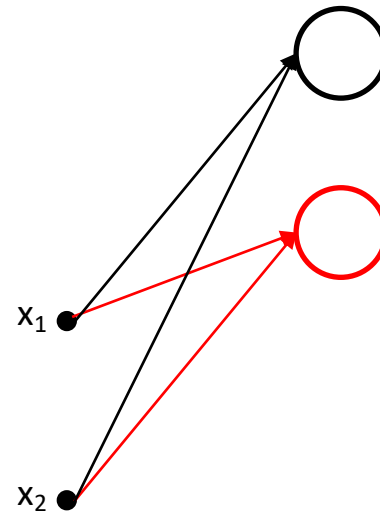
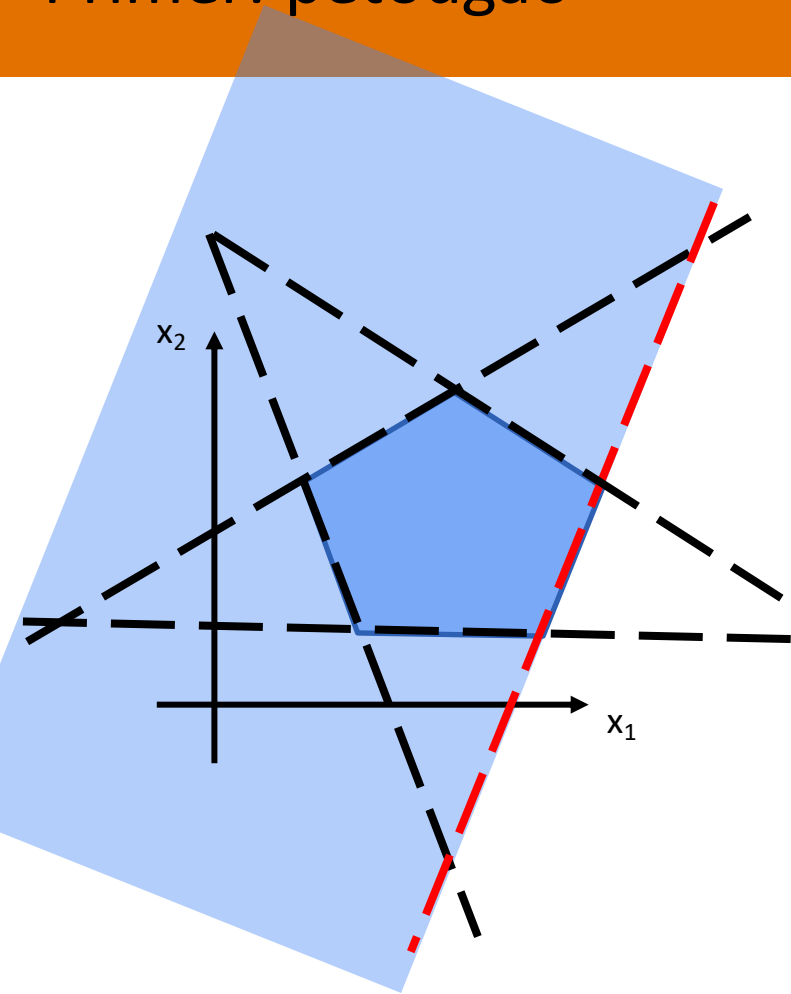
x_1 ●

x_2 ●

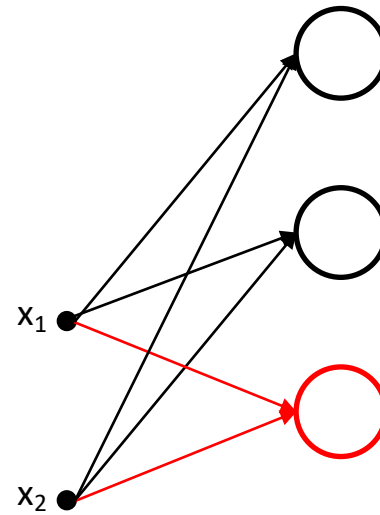
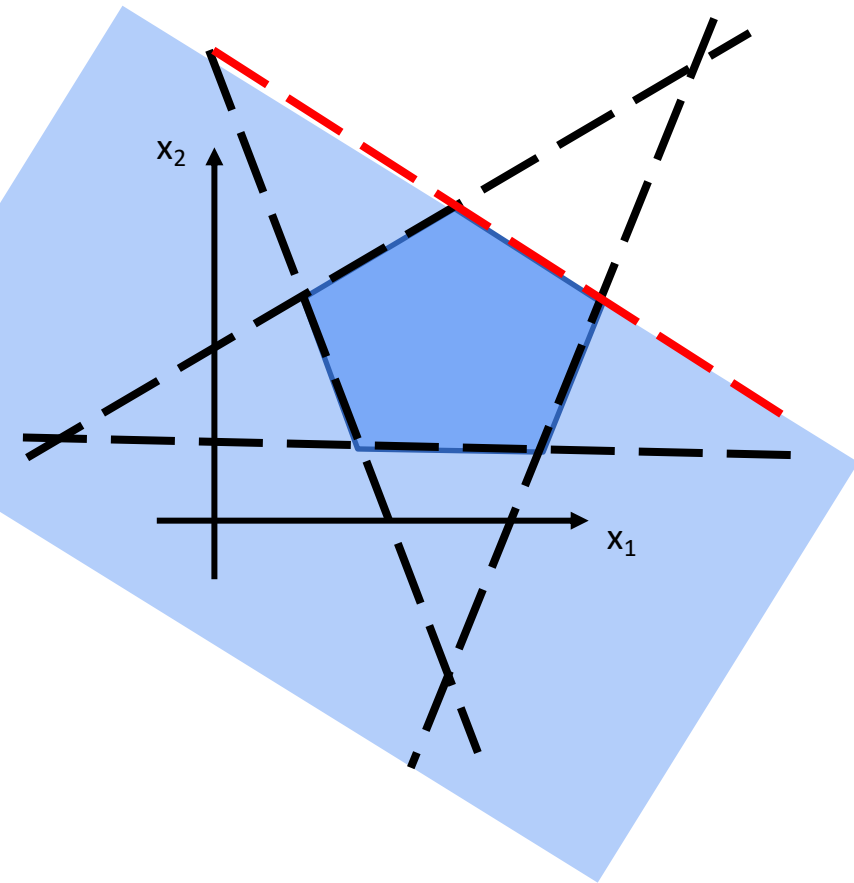
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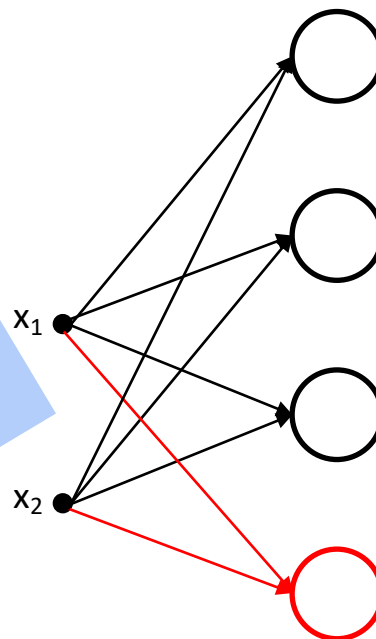
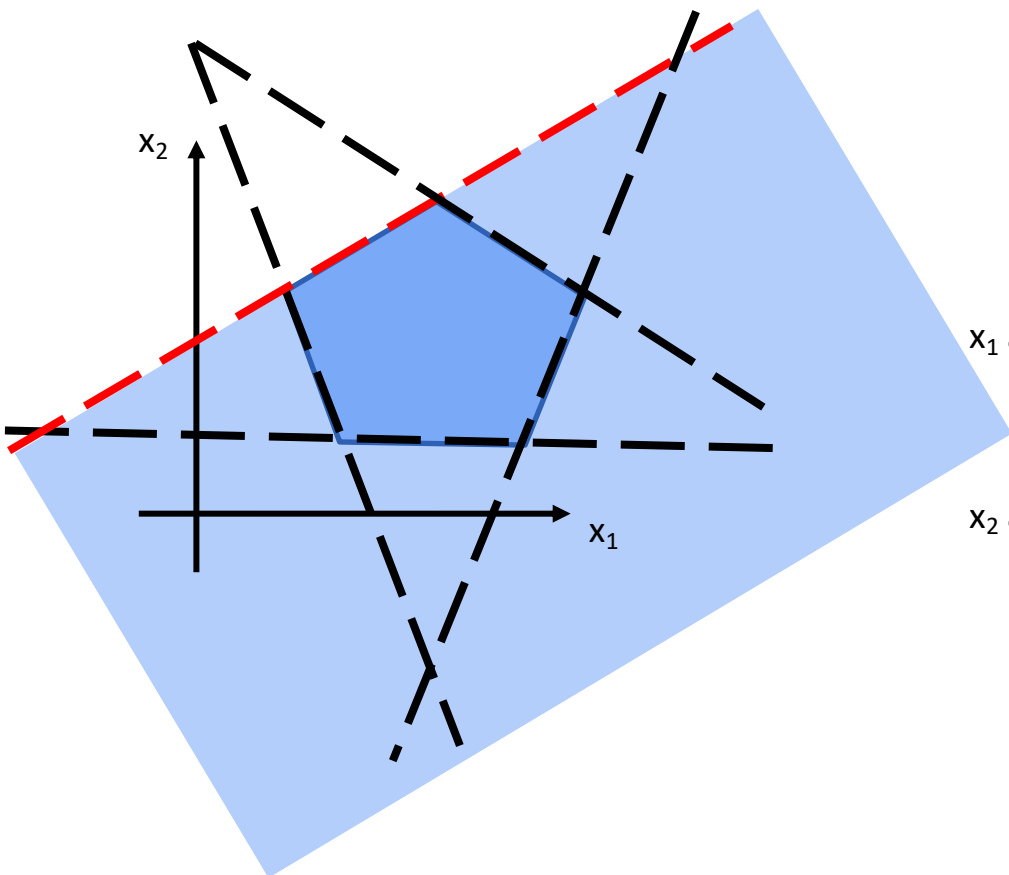
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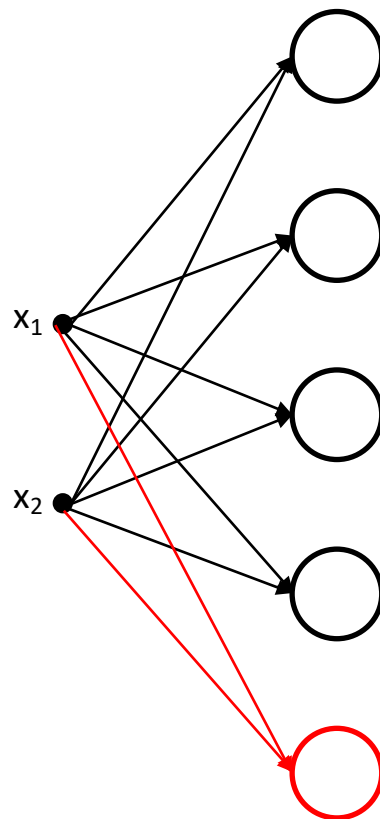
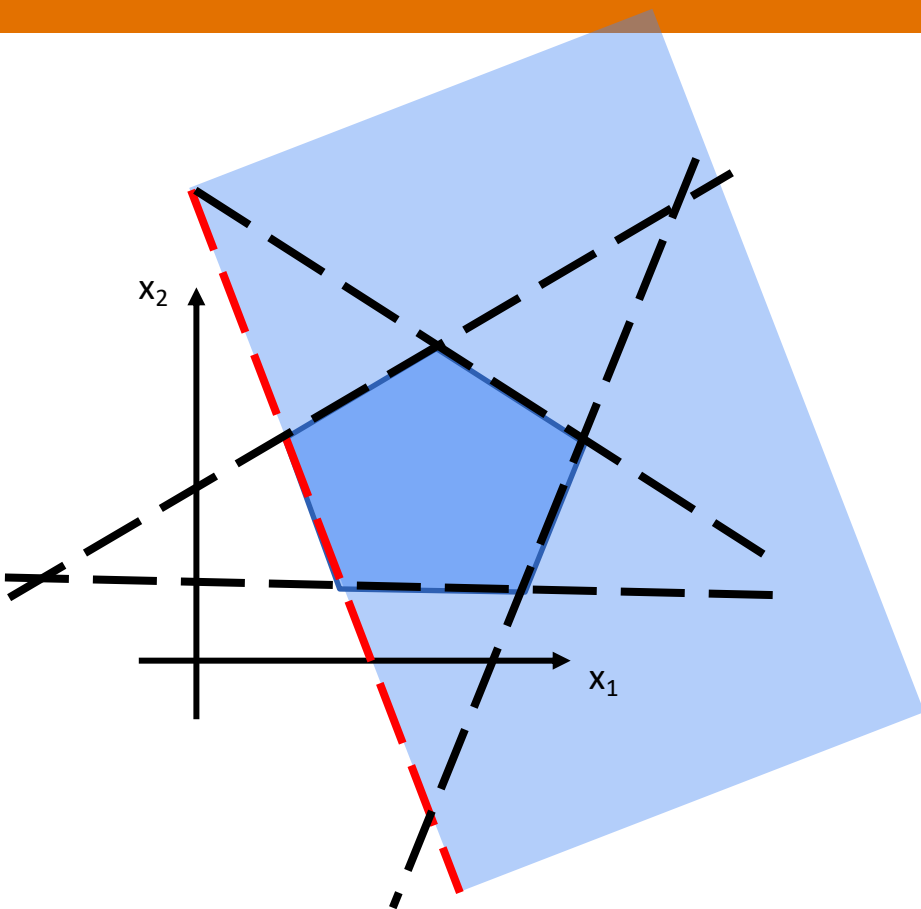
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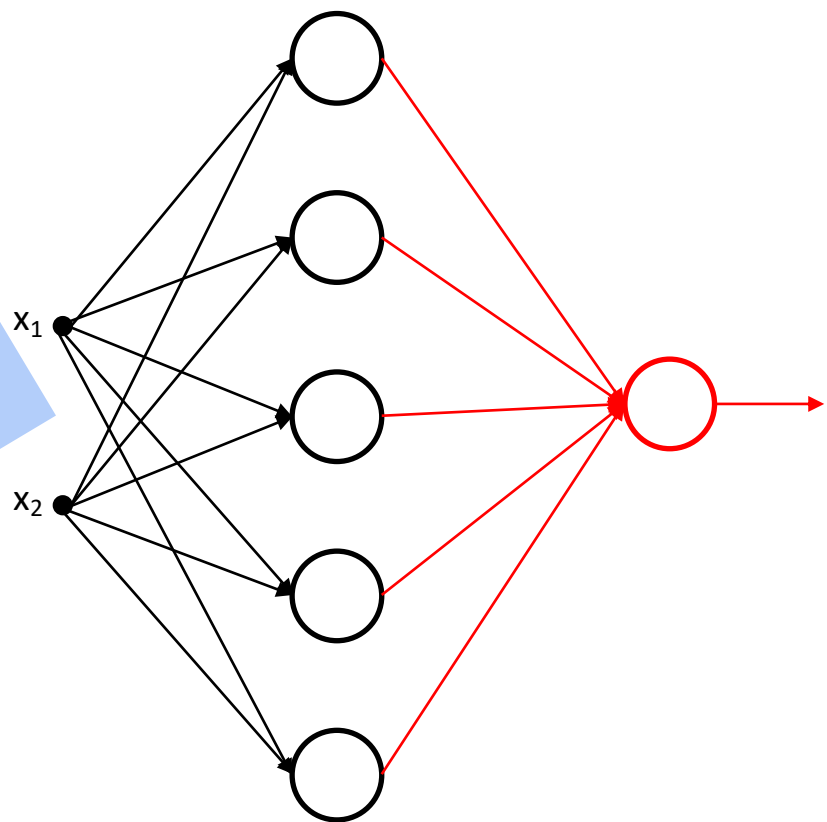
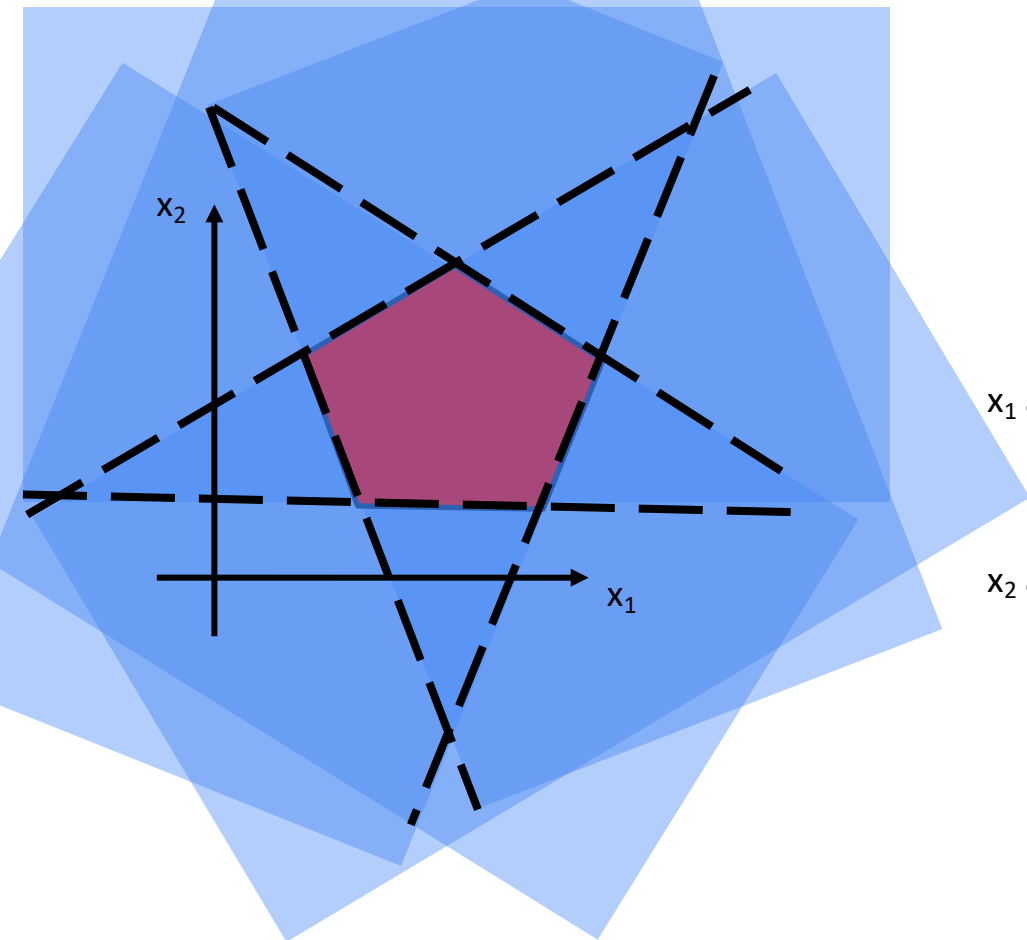
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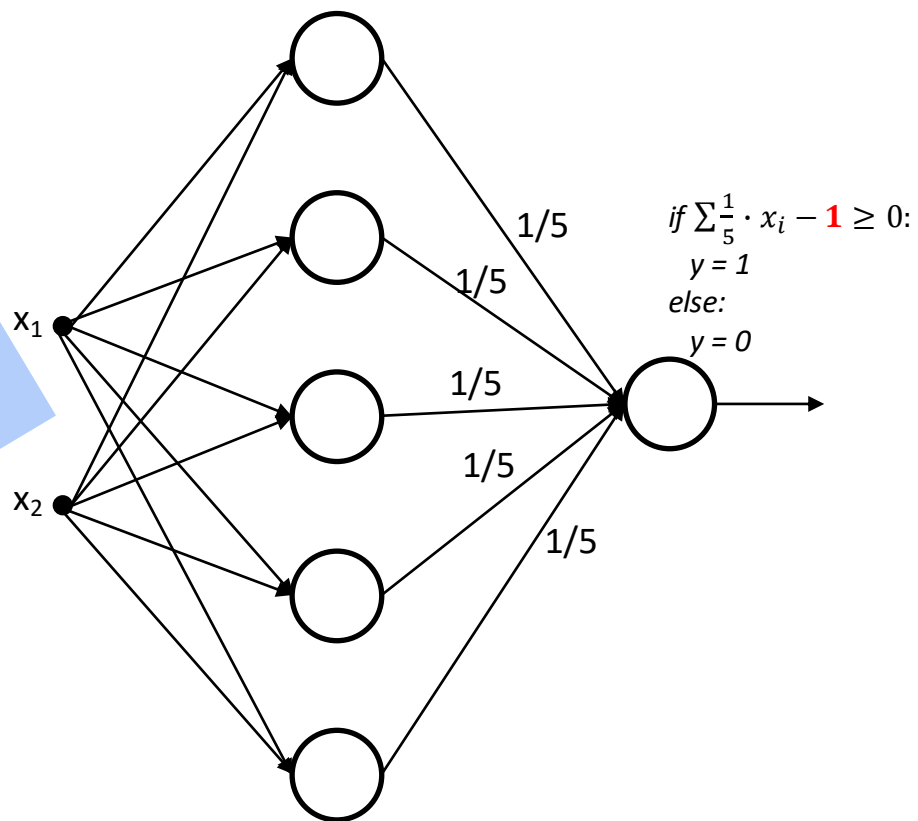
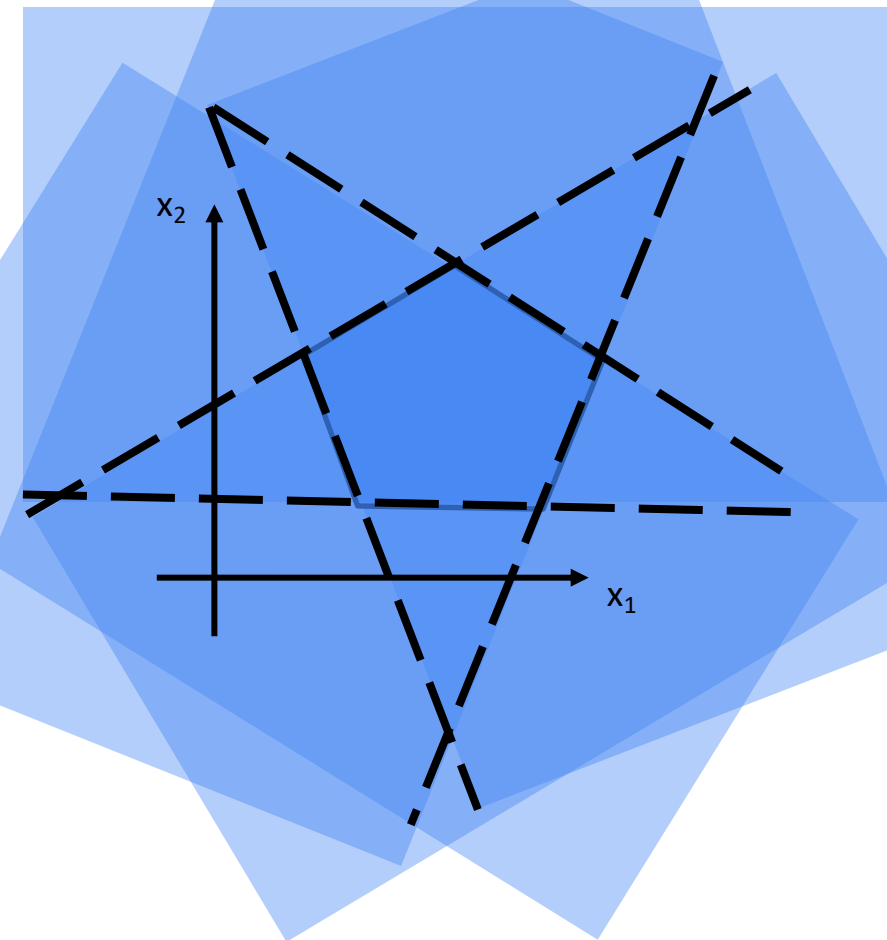
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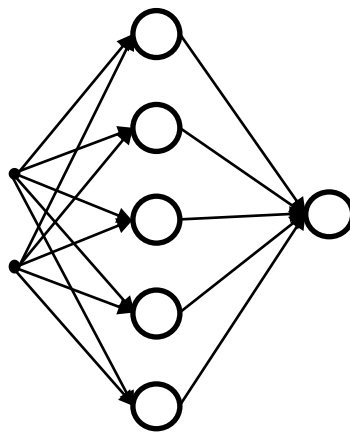
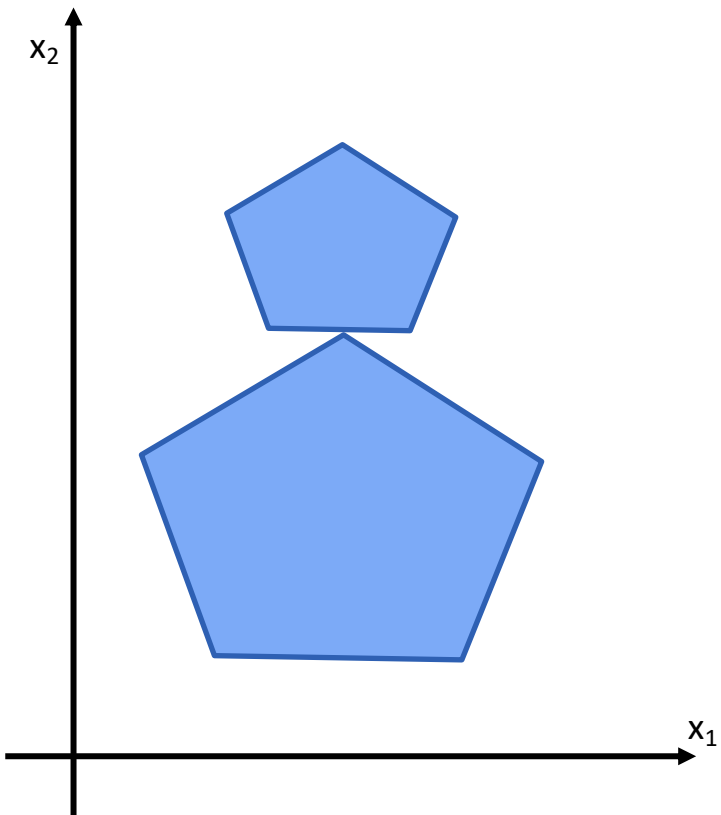


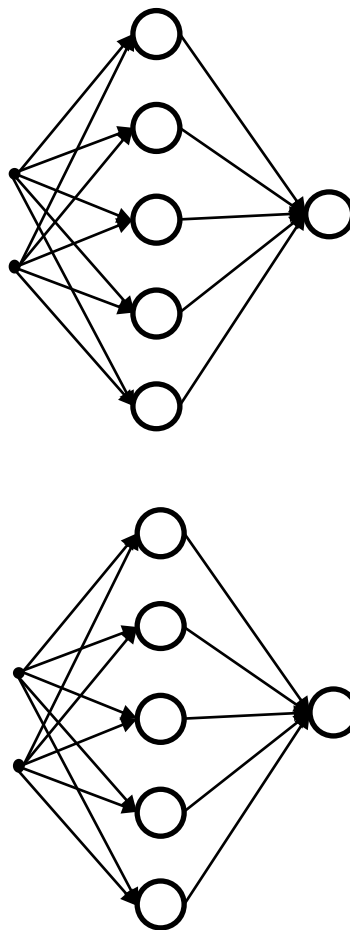
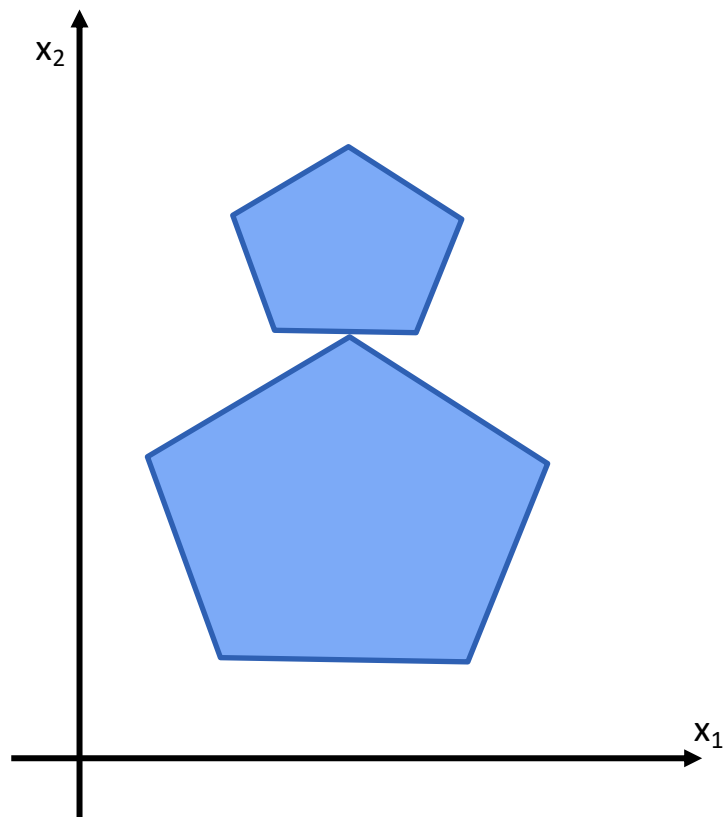
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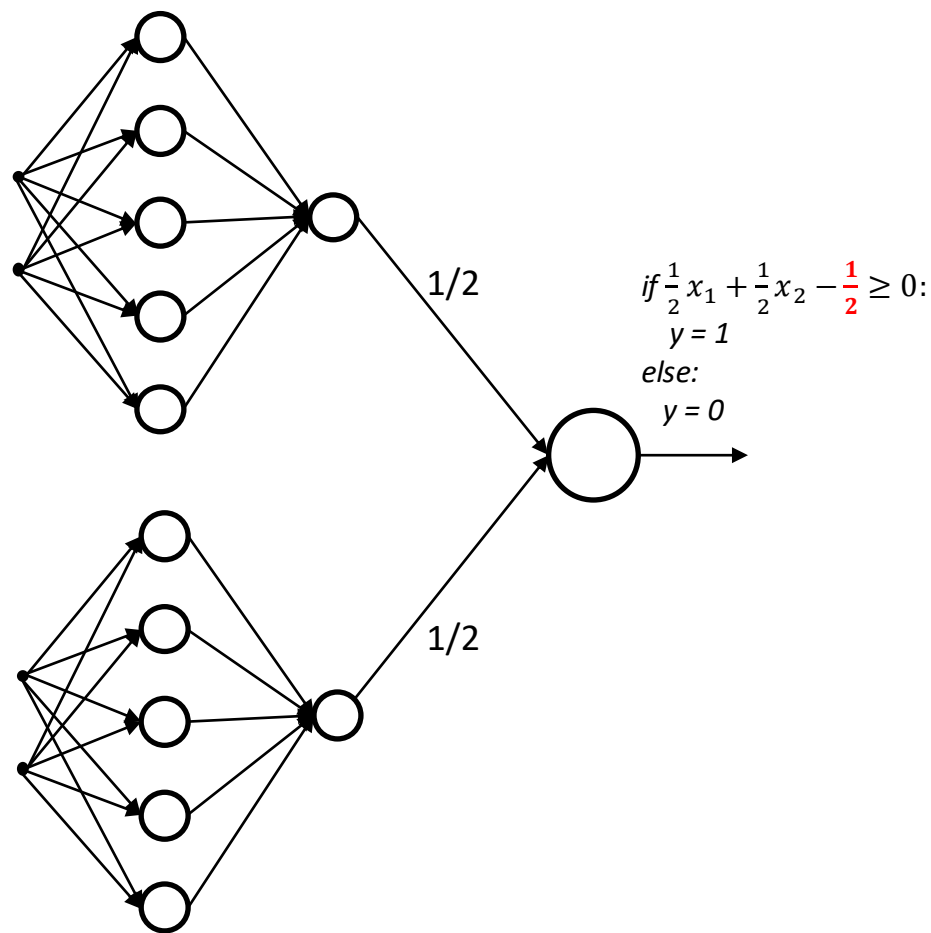
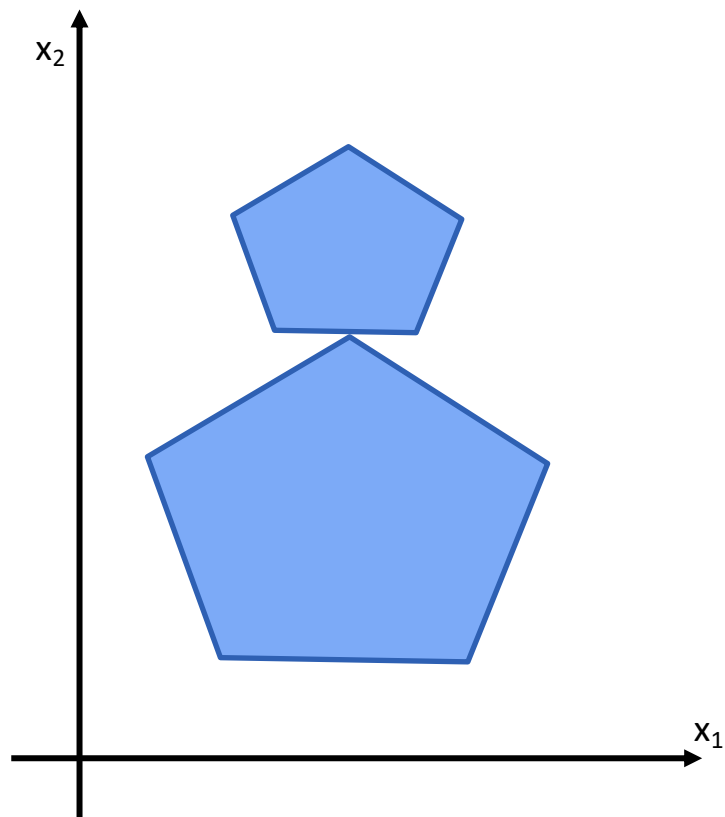


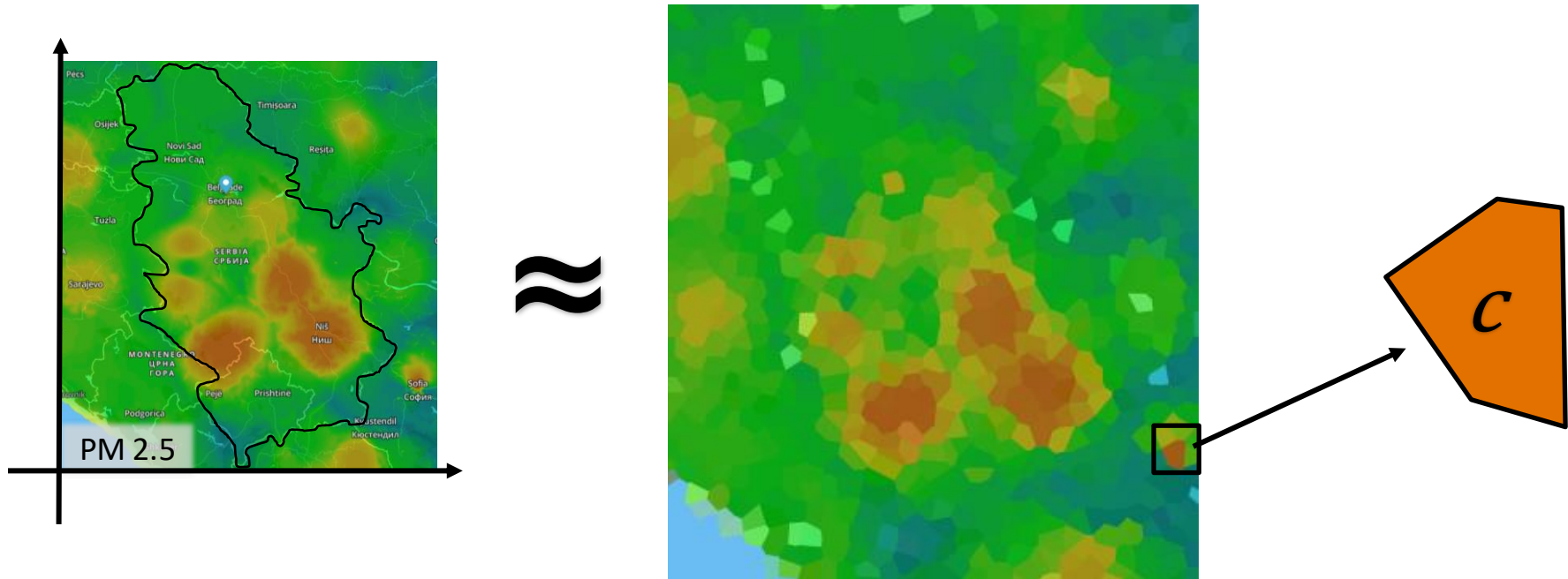
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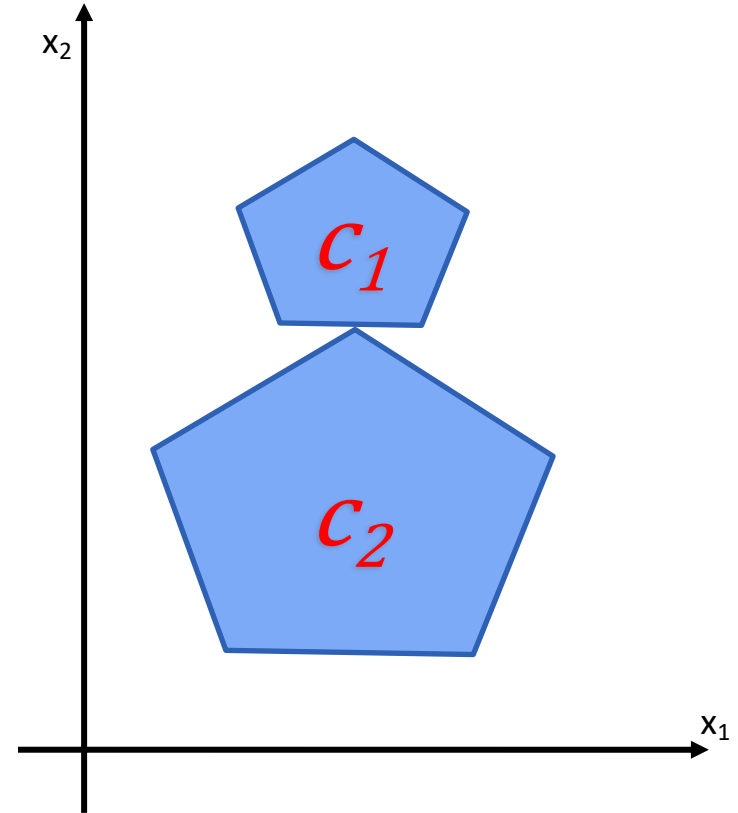
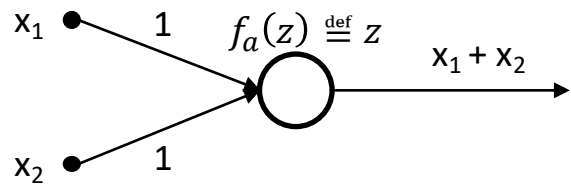


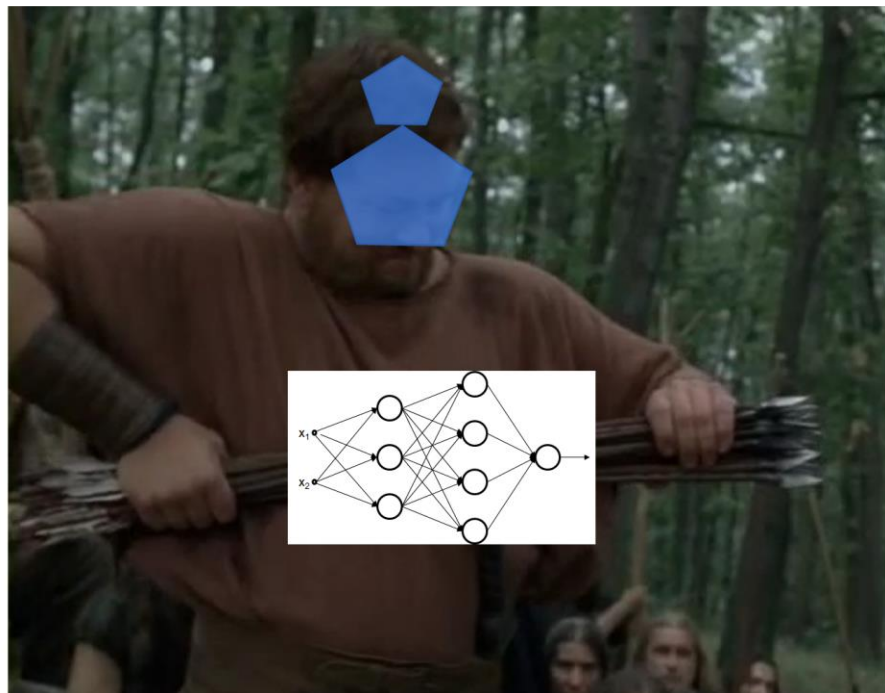
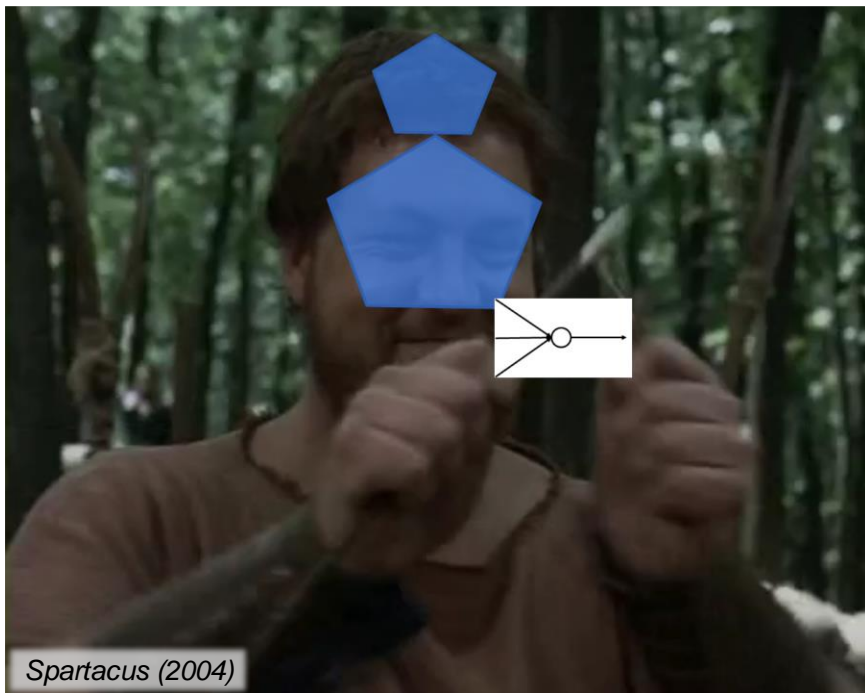




- $$f_a(\sum w_i \cdot x_i + b) \stackrel{\text{def}}{=} \begin{cases} 0, & \sum \dots < 0 \\ \mathbf{c}, & \sum \dots \geq 0 \end{cases}$$

- Neuron može i da sabira:





Hvala na pažnji!

Pitanja?