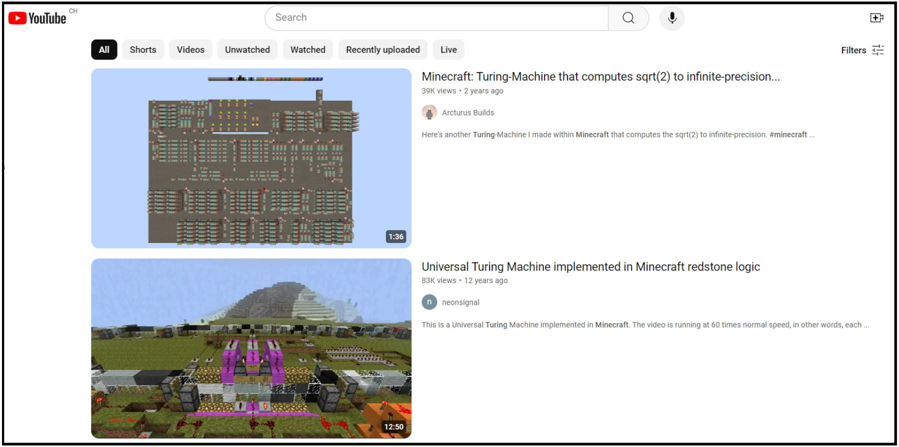
Nestandardni računari

Filip Kovačević

Matematička gimnazija

14. Maj 2024

Neočekivani računari



Magic: The Gathering is Turing Complete

Alex Churchill
Independent Researcher
Cambridge, United Kingdom
alex.churchill@cantab.net

Stella Biderman
Georgia Institute of Technology
Atlanta, United States of America
stellabiderman@gatech.edu

Austin Herrick
University of Pennsylvania
Philadelphia, United States of America
aherrick@wharton.upenn.edu

***Abstract*—*Magic: The Gathering* is a popular and famously complicated trading card game about magical combat. In this paper we show that optimal play in real-world *Magic* is at least as hard as the Halting Problem, solving a problem that has been open for a decade [1], [10]. To do this, we present a methodology for embedding an arbitrary Turing machine into a game of *Magic* such that the first player is guaranteed to win the game if and only if the Turing machine halts. Our result applies to how real *Magic* is played, can be achieved using standard-size tournament-legal decks, and does not rely on stochasticity or hidden information. Our result is also highly unusual in that all moves of both players are forced in the construction. This shows that even recognising who will win a game in which neither player has a non-trivial decision to make for the rest of the game is undecidable. We conclude with a discussion of the implications for a unified computational theory of games and remarks about the playability of such a board in a tournament setting.**

successful and highly flexible framework for modelling games as computations.

The core of this paper is the construction presented in Section IV: a universal Turing machine embedded into a game of *Magic: The Gathering*. As we can arrange for the victor of the game to be determined by the halting behaviour of the Turing machine, this construction establishes the following theorem:

Theorem 1: Determining the outcome of a game of *Magic: The Gathering* in which all remaining moves are forced is undecidable.

A. Previous Work

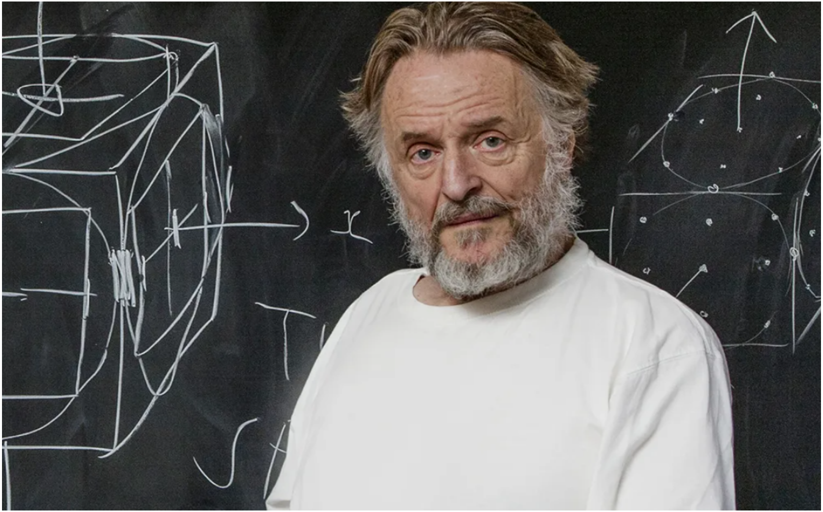
Prior to this work, no undecidable real games were known to exist. Demaine and Hearn (2009) [10] note that almost every

Koji još neočekivani modeli postoje?

Sadržaj

Sadržaj

FRACTRAN (1987)



John Horton Conway (1937-2020)

Uvod u FRACTRAN

Program: Uređena lista pozitivnih razlomaka

Program: Uređena lista pozitivnih razlomaka
Primer: $\left(\frac{455}{33}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3}\right)$

Program: Uređena lista pozitivnih razlomaka
Primer: $\left(\frac{455}{33}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3}\right)$

Ulaz: Prirodan broj n

Program: Uređena lista pozitivnih razlomaka
Primer: $\left(\frac{455}{33}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3}\right)$

Ulaz: Prirodan broj n
Primer: $n = 36$

Program: Uređena lista pozitivnih razlomaka
Primer: $\left(\frac{455}{33}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3}\right)$

Ulaz: Prirodan broj n
Primer: $n = 36$

Pravila odvijanja programa:

Program: Uređena lista pozitivnih razlomaka
Primer: $\left(\frac{455}{33}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3}\right)$

Ulaz: Prirodan broj n
Primer: $n = 36$

Pravila odvijanja programa:

1. Redom traži razlomak $\frac{p}{q}$ tako da je $\frac{p}{q} \cdot n$ prirodan broj

Program: Uređena lista pozitivnih razlomaka
Primer: $\left(\frac{455}{33}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3}\right)$

Ulaz: Prirodan broj n
Primer: $n = 36$

Pravila odvijanja programa:

1. Redom traži razlomak $\frac{p}{q}$ tako da je $\frac{p}{q} \cdot n$ prirodan broj
   Primer: $\frac{p}{q} = \frac{11}{2}$

Program: Uređena lista pozitivnih razlomaka
Primer: $\left(\frac{455}{33}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3}\right)$

Ulaz: Prirodan broj n
Primer: $n = 36$

Pravila odvijanja programa:

1. Redom traži razlomak $\frac{p}{q}$ tako da je $\frac{p}{q} \cdot n$ prirodan broj
   Primer: $\frac{p}{q} = \frac{11}{2}$
2. Ispiši $\frac{p}{q} \cdot n$, ponovi korak 1. sa novim ulazom $\frac{p}{q} \cdot n$

Uvod u FRACTRAN

Program: Uređena lista pozitivnih razlomaka
Primer: $\left(\frac{455}{33}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3}\right)$

Ulaz: Prirodan broj n
Primer: $n = 36$

Pravila odvijanja programa:

1. Redom traži razlomak $\frac{p}{q}$ tako da je $\frac{p}{q} \cdot n$ prirodan broj
   Primer: $\frac{p}{q} = \frac{11}{2}$
2. Ispiši $\frac{p}{q} \cdot n$, ponovi korak 1. sa novim ulazom $\frac{p}{q} \cdot n$
   Primer: Ispisuje $\frac{p}{q} \cdot n = 198$ i pokreće korak 1. sa tim ulazom

Uvod u FRACTRAN

Program: Uređena lista pozitivnih razlomaka
Primer: $\left(\frac{455}{33}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3}\right)$

Ulaz: Prirodan broj n
Primer: $n = 36$

Pravila odvijanja programa:

1. Redom traži razlomak $\frac{p}{q}$ tako da je $\frac{p}{q} \cdot n$ prirodan broj
   Primer: $\frac{p}{q} = \frac{11}{2}$
2. Ispiši $\frac{p}{q} \cdot n$, ponovi korak 1. sa novim ulazom $\frac{p}{q} \cdot n$
   Primer: Ispisuje $\frac{p}{q} \cdot n = 198$ i pokreće korak 1. sa tim ulazom
3. Ukoliko nijedan $\frac{p}{q} \cdot n$ nije prirodan broj, završi izvršavanje

Primer programa u FRACTRAN-u

Program: $(\frac{3}{2})$
Ulaz: $n = 36$

Primer programa u FRACTRAN-u

Program: $(\frac{3}{2})$
Ulaz: $n = 36$

Izvršavanje programa: $36 \to$

Primer programa u FRACTRAN-u

Program: $(\frac{3}{2})$
Ulaz: $n = 36$

Izvršavanje programa: $36 \rightarrow 54 \rightarrow$

Primer programa u FRACTRAN-u

Program: $\left(\frac{3}{2}\right)$
Ulaz: $n = 36$

Izvršavanje programa: $36 \rightarrow 54 \rightarrow 81$

Primer programa u FRACTRAN-u

Program: $(\frac{3}{2})$
Ulaz: $n = 36$

Izvršavanje programa: $36 \rightarrow 54 \rightarrow 81 \rightarrow$ stop

Primer programa u FRACTRAN-u

Program: $(\frac{3}{2})$
Ulaz: $n = 36$

Izvršavanje programa: $36 \to 54 \to 81 \to$ stop

Analiza izvršavanja:
$36 = 2^{R_2} 3^{R_3}$, gde $R_2 = 2$ i $R_3 = 2$.
$81 = 3^{R_2 + R_3}$

Primer programa u FRACTRAN-u

Program: $(\frac{3}{2})$
Ulaz: $n = 36$

Izvršavanje programa: $36 \to 54 \to 81 \to$ stop

Analiza izvršavanja:
$36 = 2^{R_2}3^{R_3}$, gde $R_2 = 2$ i $R_3 = 2$.
$81 = 3^{R_2+R_3}$

Vreme	Registri	
	R_2	R_3
t = 0	2	2
t = 1	1	3
t = 2	0	4
t = 3	/	/

Program: $(\frac{3}{2})$

Analiza programa:
Ukoliko $R_2 > 0 : R_2 - -\ ,\ R_3 + +$.
Ukoliko $R_2 = 0$: zaustavi.

Sabiranje u FRACTRAN-u

Program: $(\frac{3}{2})$

Analiza programa:
Ukoliko $R_2 > 0 : R_2 --\ ,\ R_3 ++$.
Ukoliko $R_2 = 0 :$ zaustavi.

Za ulaz $2^{R_2}3^{R_3}$ ispisuje $3^{R_2+R_3}$.

Program: $\left(\frac{455}{33}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3}\right)$

Množenje u FRACTRAN-u

Program: $\left(\frac{455}{33}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3}\right) = \left(\frac{5 \cdot 7 \cdot 13}{3 \cdot 11}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3}\right)$

Množenje u FRACTRAN-u

Program: $\left(\frac{455}{33}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3}\right) = \left(\frac{5\cdot7\cdot13}{3\cdot11}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3}\right)$

Ulaz: $n = 2^{R_2}3^{R_3}$
Izlaz: $5^{R_2\cdot R_3}$

Program: $\left(\frac{455}{33}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3}\right) = \left(\frac{5 \cdot 7 \cdot 13}{3 \cdot 11}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3}\right)$

Ulaz: $n = 2^{R_2} 3^{R_3}$
Izlaz: $5^{R_2 \cdot R_3}$

Pomoćni registri:
R_7: Privremeno čuvanje R_3

Množenje u FRACTRAN-u

Program: $\left(\frac{455}{33}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3}\right) = \left(\frac{5 \cdot 7 \cdot 13}{3 \cdot 11}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3}\right)$

Ulaz: $n = 2^{R_2} 3^{R_3}$
Izlaz: $5^{R_2 \cdot R_3}$

Pomoćni registri:
R_7: Privremeno čuvanje R_3
R_{11} i R_{13}: Indikatori za kontrolu petlje

Množenje u FRACTRAN-u

Ulaz: $n = 36 = 2^2 3^2$; **Program**: $\left(\frac{5\cdot 7\cdot 13}{3\cdot 11}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3}\right)$

Množenje u FRACTRAN-u

Ulaz: $n = 36 = 2^2 3^2$; **Program**: $\left(\frac{5\cdot7\cdot13}{3\cdot11}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3}\right)$

Vreme	Registri						
	R_2	R_3	R_5	R_7	R_{11}	R_{13}	Frac
t = 0	2	2	0	0	0	0	

Množenje u FRACTRAN-u

Ulaz: $n = 36 = 2^2 3^2$; **Program**: $\left(\frac{5\cdot7\cdot13}{3\cdot11}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3}\right)$

Vreme	Registri						
	R_2	R_3	R_5	R_7	R_{11}	R_{13}	Frac
t = 0	2	2	0	0	0	0	$\frac{11}{2}$
t = 1	1	2	0	0	1	0	

Ulaz: $n = 36 = 2^2 3^2$; **Program**: $\left(\frac{5\cdot7\cdot13}{3\cdot11}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3}\right)$

Vreme	Registri						
	R_2	R_3	R_5	R_7	R_{11}	R_{13}	Frac
t = 0	2	2	0	0	0	0	$\frac{11}{2}$
t = 1	1	2	0	0	1	0	$\frac{5\cdot7\cdot13}{3\cdot11}$
t = 2	1	1	1	1	0	1	

Ulaz: $n = 36 = 2^2 3^2$; **Program**: $\left(\frac{5 \cdot 7 \cdot 13}{3 \cdot 11}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3}\right)$

Vreme	Registri						
	R_2	R_3	R_5	R_7	R_{11}	R_{13}	Frac
t = 0	2	2	0	0	0	0	$\frac{11}{2}$
t = 1	1	2	0	0	1	0	$\frac{5 \cdot 7 \cdot 13}{3 \cdot 11}$
t = 2	1	1	1	1	0	1	$\frac{11}{13}$
t = 3	1	1	1	1	1	0	

Ulaz: $n = 36 = 2^2 3^2$; **Program**: $\left(\frac{5\cdot7\cdot13}{3\cdot11}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3}\right)$

Vreme	Registri						
	R_2	R_3	R_5	R_7	R_{11}	R_{13}	Frac
t = 0	2	2	0	0	0	0	$\frac{11}{2}$
t = 1	1	2	0	0	1	0	$\frac{5\cdot7\cdot13}{3\cdot11}$
t = 2	1	1	1	1	0	1	$\frac{11}{13}$
t = 3	1	1	1	1	1	0	$\frac{5\cdot7\cdot13}{3\cdot11}$
t = 4	1	0	2	2	0	1	

Množenje u FRACTRAN-u

Ulaz: $n = 36 = 2^2 3^2$; **Program**: $\left(\frac{5\cdot7\cdot13}{3\cdot11}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3}\right)$

Vreme	Registri						
	R_2	R_3	R_5	R_7	R_{11}	R_{13}	Frac
t = 0	2	2	0	0	0	0	$\frac{11}{2}$
t = 1	1	2	0	0	1	0	$\frac{5\cdot7\cdot13}{3\cdot11}$
t = 2	1	1	1	1	0	1	$\frac{11}{13}$
t = 3	1	1	1	1	1	0	$\frac{5\cdot7\cdot13}{3\cdot11}$
t = 4	1	0	2	2	0	1	$\frac{11}{13}$
t = 5	1	0	2	2	1	0	

Množenje u FRACTRAN-u

Ulaz: $n = 36 = 2^2 3^2$; **Program**: $\left(\frac{5\cdot7\cdot13}{3\cdot11}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3}\right)$

Vreme	Registri						
	R_2	R_3	R_5	R_7	R_{11}	R_{13}	Frac
t = 0	2	2	0	0	0	0	$\frac{11}{2}$
t = 1	1	2	0	0	1	0	$\frac{5\cdot7\cdot13}{3\cdot11}$
t = 2	1	1	1	1	0	1	$\frac{11}{13}$
t = 3	1	1	1	1	1	0	$\frac{5\cdot7\cdot13}{3\cdot11}$
t = 4	1	0	2	2	0	1	$\frac{11}{13}$
t = 5	1	0	2	2	1	0	$\frac{1}{11}$
t = 6	1	0	2	2	0	0	

Ulaz: $n = 36 = 2^2 3^2$; **Program**: $\left(\frac{5\cdot7\cdot13}{3\cdot11}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3}\right)$

Vreme	Registri						
	R_2	R_3	R_5	R_7	R_{11}	R_{13}	Frac
t = 0	2	2	0	0	0	0	$\frac{11}{2}$
t = 1	1	2	0	0	1	0	$\frac{5\cdot7\cdot13}{3\cdot11}$
t = 2	1	1	1	1	0	1	$\frac{11}{13}$
t = 3	1	1	1	1	1	0	$\frac{5\cdot7\cdot13}{3\cdot11}$
t = 4	1	0	2	2	0	1	$\frac{11}{13}$
t = 5	1	0	2	2	1	0	$\frac{1}{11}$
t = 6	1	0	2	2	0	0	$\frac{3}{7}$
t = 7	1	1	2	1	0	0	

Množenje u FRACTRAN-u

Ulaz: $n = 36 = 2^2 3^2$; **Program**: $\left(\frac{5 \cdot 7 \cdot 13}{3 \cdot 11}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3}\right)$

Vreme	Registri						
	R_2	R_3	R_5	R_7	R_{11}	R_{13}	Frac
t = 0	2	2	0	0	0	0	$\frac{11}{2}$
t = 1	1	2	0	0	1	0	$\frac{5 \cdot 7 \cdot 13}{3 \cdot 11}$
t = 2	1	1	1	1	0	1	$\frac{11}{13}$
t = 3	1	1	1	1	1	0	$\frac{5 \cdot 7 \cdot 13}{3 \cdot 11}$
t = 4	1	0	2	2	0	1	$\frac{11}{13}$
t = 5	1	0	2	2	1	0	$\frac{1}{11}$
t = 6	1	0	2	2	0	0	$\frac{3}{7}$
t = 7	1	1	2	1	0	0	$\frac{3}{7}$
t = 8	1	2	2	0	0	0	

Program: $\left(\frac{17}{91}, \frac{78}{85}, \frac{19}{51}, \frac{23}{38}, \frac{29}{33}, \frac{77}{29}, \frac{95}{23}, \frac{77}{19}, \frac{1}{17}, \frac{11}{13}, \frac{13}{11}, \frac{15}{2}, \frac{1}{7}, \frac{55}{1}\right)$

Program: $\left(\frac{17}{91}, \frac{78}{85}, \frac{19}{51}, \frac{23}{38}, \frac{29}{33}, \frac{77}{29}, \frac{95}{23}, \frac{77}{19}, \frac{1}{17}, \frac{11}{13}, \frac{13}{11}, \frac{15}{2}, \frac{1}{7}, \frac{55}{1}\right)$

Ulaz: $n = 2$
Izlaz: Stepeni dvojke čiji su eksponenti prosti brojevi :

$$2^2, 2^3, 2^5, 2^7, 2^{11}, 2^{13}, 2^{17}, 2^{19}, \ldots,$$

i to redom po veličini.

Program:

$$\frac{365}{46}\ \frac{29}{161}\ \frac{79}{575}\ \frac{679}{451}\ \frac{3159}{413}\ \frac{83}{407}\ \frac{473}{371}\ \frac{638}{355}\ \frac{434}{335}\ \frac{89}{235}\ \frac{17}{209}\ \frac{79}{122}$$

$$\frac{31}{183}\ \frac{41}{115}\ \frac{517}{89}\ \frac{111}{83}\ \frac{305}{79}\ \frac{23}{73}\ \frac{73}{71}\ \frac{61}{67}\ \frac{37}{61}\ \frac{19}{59}\ \frac{89}{57}\ \frac{41}{53}\ \frac{833}{47}\ \frac{53}{43}$$

$$\frac{86}{41}\ \frac{13}{38}\ \frac{23}{37}\ \frac{67}{31}\ \frac{71}{29}\ \frac{83}{19}\ \frac{475}{17}\ \frac{59}{13}\ \frac{41}{291}\ \frac{1}{7}\ \frac{1}{11}\ \frac{1}{1024}\ \frac{1}{97}\ \frac{89}{1}$$

Generator decimala broja π

Program:

$$\frac{365}{46}\ \frac{29}{161}\ \frac{79}{575}\ \frac{679}{451}\ \frac{3159}{413}\ \frac{83}{407}\ \frac{473}{371}\ \frac{638}{355}\ \frac{434}{335}\ \frac{89}{235}\ \frac{17}{209}\ \frac{79}{122}$$

$$\frac{31}{183}\ \frac{41}{115}\ \frac{517}{89}\ \frac{111}{83}\ \frac{305}{79}\ \frac{23}{73}\ \frac{73}{71}\ \frac{61}{67}\ \frac{37}{61}\ \frac{19}{59}\ \frac{89}{57}\ \frac{41}{53}\ \frac{833}{47}\ \frac{53}{43}$$

$$\frac{86}{41}\ \frac{13}{38}\ \frac{23}{37}\ \frac{67}{31}\ \frac{71}{29}\ \frac{83}{19}\ \frac{475}{17}\ \frac{59}{13}\ \frac{41}{291}\ \frac{1}{7}\ \frac{1}{11}\ \frac{1}{1024}\ \frac{1}{97}\ \frac{89}{1}$$

Ulaz: 2^n
Izlaz: $2^{\pi(n)}$, gde je $\pi(n)$ n-ta decimala broja π.

Generator decimala broja π

Program:

$$\frac{365}{46}\ \frac{29}{161}\ \frac{79}{575}\ \frac{679}{451}\ \frac{3159}{413}\ \frac{83}{407}\ \frac{473}{371}\ \frac{638}{355}\ \frac{434}{335}\ \frac{89}{235}\ \frac{17}{209}\ \frac{79}{122}$$

$$\frac{31}{183}\ \frac{41}{115}\ \frac{517}{89}\ \frac{111}{83}\ \frac{305}{79}\ \frac{23}{73}\ \frac{73}{71}\ \frac{61}{67}\ \frac{37}{61}\ \frac{19}{59}\ \frac{89}{57}\ \frac{41}{53}\ \frac{833}{47}\ \frac{53}{43}$$

$$\frac{86}{41}\ \frac{13}{38}\ \frac{23}{37}\ \frac{67}{31}\ \frac{71}{29}\ \frac{83}{19}\ \frac{475}{17}\ \frac{59}{13}\ \frac{41}{291}\ \frac{1}{7}\ \frac{1}{11}\ \frac{1}{1024}\ \frac{1}{97}\ \frac{89}{1}$$

Ulaz: 2^n
Izlaz: $2^{\pi(n)}$, gde je $\pi(n)$ n-ta decimala broja π.

n =	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	...
$\pi(n)$ =	3	1	4	1	5	9	2	6	5	3	5	8	9	7	9	3	2	3	8	4	6	...

FRACTRAN je računski univerzalan

Teorema (Conway 1987)

FRACTRAN je računski univerzalan (eng. Turing complete), odnosno može simulirati bilo koji računar opšte namene.

Sadržaj

Definicija

Diofantska jednačina je je polinomijalni izraz $p(x_1, \ldots, x_n) = 0$ čiji su koeficijenti i tražena rešenja celi brojevi.

Definicija

Diofantska jednačina je je polinomijalni izraz $p(x_1, \ldots, x_n) = 0$ čiji su koeficijenti i tražena rešenja celi brojevi.

Primeri:

$$39x_1^2 + 39x_2^2 - 195 = 0$$

$$x_1^3 + x_2^3 + x_3^3 - 42 = 0$$

$$x_1^7 + x_2^7 - x_3^7 = 0$$

Definicija

Diofantska jednačina je je polinomijalni izraz $p(x_1, \ldots, x_n) = 0$ čiji su koeficijenti i tražena rešenja celi brojevi.

Primeri:

$$39x_1^2 + 39x_2^2 - 195 = 0$$
$$x_1^3 + x_2^3 + x_3^3 - 42 = 0$$
$$x_1^7 + x_2^7 - x_3^7 = 0$$

Komentar

Iako u kompleksnim brojevima jadnačina uvek ima rešenja, u celim to nije nužno tačno.

Diofantski računar

Program: Diofantska jednačina $p(t, x_1, \ldots, x_n) = 0$

Program: Diofantska jednačina $p(t, x_1, \ldots, x_n) = 0$

Izlaz: Skup D_p, sadrži tačno sva rešenja jednačine po t:

$$D_p = \left\{ t \in \mathbb{Z} \;\middle|\; \exists x_1, \ldots, x_n \; t.d. \; p(t, x_1, \ldots, x_n) = 0 \right\}$$

Program: Diofantska jednačina $p(t, x_1, \ldots, x_n) = 0$

Izlaz: Skup D_p, sadrži tačno sva rešenja jednačine po t:

$$D_p = \left\{ t \in \mathbb{Z} \;\middle|\; \exists x_1, \ldots, x_n \ t.d.\ p(t, x_1, \ldots, x_n) = 0 \right\}$$

Primer:
Program: $p(t, x) = t - x^2$

Diofantski računar

Program: Diofantska jednačina $p(t, x_1, \ldots, x_n) = 0$

Izlaz: Skup D_p, sadrži tačno sva rešenja jednačine po t:

$$D_p = \left\{ t \in \mathbb{Z} \;\middle|\; \exists x_1, \ldots, x_n \; t.d. \; p(t, x_1, \ldots, x_n) = 0 \right\}$$

Primer:
Program: $p(t, x) = t - x^2$
Izlaz: Skup svih kvadrata $D_p = \{1, 4, 9, 16, \ldots\}$

Primeri diofantskih programa

1. **Skup svih prirodnih brojeva**:

1. Skup svih prirodnih brojeva:

Teorema (Lagranž 1770)

Svaki prirodan broj t se može predstaviti kao zbir četiri celobrojna kvadrata:

$$t = x_1^2 + x_2^2 + x_3^2 + x_4^2$$

1. Skup svih prirodnih brojeva:

Teorema (Lagranž 1770)

Svaki prirodan broj t se može predstaviti kao zbir četiri celobrojna kvadrata:

$$t = x_1^2 + x_2^2 + x_3^2 + x_4^2$$

Program: $p(t, x_1, x_2, x_3, x_4) = t - x_1^2 - x_2^2 - x_3^2 - x_4^2$

1. Skup svih prirodnih brojeva:

Teorema (Lagranž 1770)

Svaki prirodan broj t se može predstaviti kao zbir četiri celobrojna kvadrata:

$$t = x_1^2 + x_2^2 + x_3^2 + x_4^2$$

Program: $p(t, x_1, x_2, x_3, x_4) = t - x_1^2 - x_2^2 - x_3^2 - x_4^2$

2. Skup svih brojeva manjih od zadatog n:

1. Skup svih prirodnih brojeva:

Teorema (Lagranž 1770)

Svaki prirodan broj t se može predstaviti kao zbir četiri celobrojna kvadrata:

$$t = x_1^2 + x_2^2 + x_3^2 + x_4^2$$

Program: $p(t, x_1, x_2, x_3, x_4) = t - x_1^2 - x_2^2 - x_3^2 - x_4^2$

2. Skup svih brojeva manjih od zadatog n:

Program: $p(t, x_1, x_2, x_3, x_4) = (n-1) - t - x_1^2 - x_2^2 - x_3^2 - x_4^2$

3. Skup svih brojeva koji nisu kvadrati:

3. Skup svih brojeva koji nisu kvadrati:

Ideja: t nije kvadrat kada $n^2 < t < (n+1)^2$

3. Skup svih brojeva koji nisu kvadrati:

Ideja: t nije kvadrat kada $n^2 < t < (n+1)^2$

Program:

$$\begin{aligned} p(t, x_1, x_2, x_3, \ldots, x_{11}) = &\ (t - x_1^2 - 1 - x_2)^2 \\ &+ (t - x_1^2 - 2x_1 - 1 + x_3 + 1)^2 \\ &+ (x_2 \text{ prirodan broj})^2 \\ &+ (x_3 \text{ prirodan broj})^2 \end{aligned}$$

3. Skup svih brojeva koji nisu kvadrati:

Ideja: t nije kvadrat kada $n^2 < t < (n+1)^2$

Program:

$$\begin{aligned} p(t, x_1, x_2, x_3, \ldots, x_{11}) = & (t - x_1^2 - 1 - x_2)^2 \\ & + (t - x_1^2 - 2x_1 - 1 + x_3 + 1)^2 \\ & + (x_2 \text{ prirodan broj})^2 \\ & + (x_3 \text{ prirodan broj})^2 \end{aligned}$$

4. Skup svih složenih brojeva:

3. Skup svih brojeva koji nisu kvadrati:

Ideja: t nije kvadrat kada $n^2 < t < (n+1)^2$

Program:

$$\begin{aligned} p(t, x_1, x_2, x_3, \ldots, x_{11}) = & \, (t - x_1^2 - 1 - x_2)^2 \\ & + (t - x_1^2 - 2x_1 - 1 + x_3 + 1)^2 \\ & + (x_2 \text{ prirodan broj})^2 \\ & + (x_3 \text{ prirodan broj})^2 \end{aligned}$$

4. Skup svih složenih brojeva:

Program: $p(t, x_1, x_2) = t - (x_1 + 2)(x_2 + 2), x_1, x_2$ prirodni

5. Skup svih prostih brojeva, $D_p = \{2, 3, 5, 7, 11, \dots\}$:

5. Skup svih prostih brojeva, $D_p = \{2, 3, 5, 7, 11, \dots\}$:

Program: ???

5. Skup svih prostih brojeva, $D_p = \{2, 3, 5, 7, 11, \ldots\}$:

Program: ???

6. Skup svih cifara broja π, $D_p = \{3, 14, 159, 2653, 5897, \ldots\}$:

5. Skup svih prostih brojeva, $D_p = \{2, 3, 5, 7, 11, \ldots\}$:

Program: ???

6. Skup svih cifara broja π, $D_p = \{3, 14, 159, 2653, 5897, \ldots\}$:

Program: ???

Diofantski računar je računski univerzalan

Stav

Svaki diofantski program se može simulirati bilo kojim univerzalnim računarom.

Diofantski računar je računski univerzalan

Stav

Svaki diofantski program se može simulirati bilo kojim univerzalnim računarom.

Ideja: Probaj redom elemente iz $\mathbb{Z}^{n+1}$ u jednačini $p(t, x_1 \ldots, x_n)$ i ispiši t koji su rešenja.

Stav

Svaki diofantski program se može simulirati bilo kojim univerzalnim računarom.

Ideja: Probaj redom elemente iz $\mathbb{Z}^{n+1}$ u jednačini $p(t, x_1 \ldots, x_n)$ i ispiši t koji su rešenja.

Teorema (MRDP: Matijašević-Robinson-Davis-Putnam, 1970)

Sve što se može ispisati univerzalnim računarom se može simulirati diofantskim programom.

Interesantne posledice MRDP

Skup svih prostih brojeva, $D_k = \{2, 3, 5, 7, 11, \dots\}$:

Program: Postoji!

Interesantne posledice MRDP

Skup svih prostih brojeva, $D_k = \{2, 3, 5, 7, 11, \dots\}$:

Program: Postoji!

$$
\begin{aligned}
(k+2)\{1 &- [wx + h + j - q]^2 \\
&- [(gk + 2g + k + 1)(h + j) + h - z]^2 \\
&- [2n + p + q + z - e]^2 \\
&- [16(k+1)^3(k+2)(n+1)^2 + 1 - f^2]^2 \\
&- [e^3(e+2)(a+1)^2 + 1 - o^2]^2 \\
&- [(a^2 - 1)y^2 + 1 - x^2]^2 \\
&- [16r^2y^4(a^2 - 1) + 1 - u^2]^2 \\
&- [n + l + v - y]^2 \\
&- [((a + u^2(u2 - a))^2 - 1)(n + 4dy)^2 + 1 - (x + cu)^2]^2 \\
&- [(a^1 - 1)l^2 + 1 - m^2]^2 \\
&- [q + y(a - p - 1) + s(2ap + 2a - p^2 - 2p - 2) - x]^2 \\
&- [z + pl(a - p) + t(2ap - p^2 - 1) - pm]^2 \\
&- [ai + k + 1 - l - i]^2 \\
&- [p + l(a - n - 1) + b(2an + 2a - n^2 - 2n - n) - m]^2\}
\end{aligned}
$$

Interesantne posledice MRDP

Posledica (Rešenje 10. Hilbertovog problema)

Ne postoji algoritam koji može da rešava proizvoljnu diofantsku jednačinu.

Interesantne posledice MRDP

Posledica (Rešenje 10. Hilbertovog problema)

Ne postoji algoritam koji može da rešava proizvoljnu diofantsku jednačinu.

Ideja:

PPS $\rightarrow$ algoritam testira zaustavljanje proizvoljnog računaraskog program.

Interesantne posledice MRDP

Posledica (Rešenje 10. Hilbertovog problema)

Ne postoji algoritam koji može da rešava proizvoljnu diofantsku jednačinu.

Ideja:
PPS → algoritam testira zaustavljanje proizvoljnog računaraskog program.
Dokazano da ne postoji, tkz. problem zaustavljanja (eng. Halting problem).

Pouke

Pouke

Pouka 1

Jednostavni modeli mogu imati kompleksno ponašanje.

Pouke

Pouka 1

Jednostavni modeli mogu imati kompleksno ponašanje.

Pouka 2

Diofantske jednačine su teške.

Hvala na pažnji!

Primer teške diofantske jednačine

Da li sledeća jednačina ima rešenja? (1954)
$x^3 + y^3 + z^3 = 42$

Primer teške diofantske jednačine

Da li sledeća jednačina ima rešenja? (1954)
$x^3 + y^3 + z^3 = 42$

Najmanja rešenja (2019):
$x = -80538738812075974$
$y = 80435758145817515$
$z = 12602123297335631$

Link pregleda rezultata.

Literatura i kontakt

Hilbertov 10. problem i diofantski računar:
1. Richardson, Kyle. "Number Theory Meets Computability Theory." (2020).
2. Poonen, Bjorn. "Undecidability in Number Theory." (2008).
3. Pasten, Hector "Diophantine equations and why they are hard", (2019)

FRACTRAN:
4. Conway, John H. "Fractran: A simple universal programming language for arithmetic." (1987)
5. A gentle intro to FRACTRAN (2017):
https://esoteric.codes/blog/an-intro-to-fractran
6. Wikipedia article on FRACTRAN
https://en.wikipedia.org/wiki/FRACTRAN

Kontakt email:
filip55555@gmail.com